List of Abstracts – Talks

Waring decompositions of real binary forms and Brion's formula

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Waring's Problem over polynomial rings studies the decomposition of a homogeneous polynomial p of degree d in n variables as a linear combination of d-th powers of linear forms with coefficients in a field K. Such an expression of p is called a *Waring decomposition* (WD) of that form and when we take r minimal with this property, we call r *the Waring rank* of p over K.

This type of decompositions has aroused great interest over the last few decades because of its many applications in Mathematics, Engineering, Physics and even areas of such recent development as Data Mining or Machine Learning. In particular, we study the case $K = \mathbb{R}$ and $n = 2$. In the **Workshop de Jóvenes Investigadores 2016**, I presented an algorithm that allowed to find WD of minimal length for a real minimal form. Now, we have improved this algorithm so we will show these improvements.

As a second line of research, we will present the problem of how to efficiently compute the exact value of the integral of a polynomial $f \in \mathbb{Q}[x_1, \ldots, x_n]$ over a d-dimensional simplex Δ in $\mathbb{R}^n$. The measure considered is the integral Lebesgue measure, that we will denote by dm and the integral we want to compute by $\int_\Delta f\, dm$. One of the methods to compute the above integral is by means of a WD of the polynomial f. Then, the essential problem is to compute the d-th power of any linear form on the simplex Δ. We will show how to **effectively compute** integrals of polynomials over a finite triangulation by means of our algorithm to compute WD. Also a parametric version of Brion's formula will be presented. The techniques are based on the study of families of semialgebraic sets associated with p, the given real binary form.

Perturbation of the domain in eigenvalue problems and nonlinear evolution PDE's

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In this talk we will analyse how the perturbation of the domain affects the behavior of the eigenvalues and eigenfunctions of certain operators like the Laplace operator with Neumann boundary conditions, and the behavior of the solutions of some nonlinear reaction diffusion equations posed in these domains. We will consider some important cases of domain perturbation like a) the case where the boundary is perturbed, b) the dumbbell-type domains which consists of two fixed domains joined by a thin channel c) the case of thin domains which degenerate into a line segment, among others.

¿Qué sabemos sobre el género imaginario de un grupo?

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Las superficies de Klein no orientables y sin borde, también llamadas superficies de Riemann sin borde, son superficies compactas, sin borde y no orientables dotadas de una estructura dianalítica. Cada grupo finito G actúa como grupo de automorfismos de una de estas superficies. El mínimo género topológico de ellas se llama género imaginario del grupo G.

En los últimos años se ha llevado a cabo un gran avance en este parámetro, ya que es del que menos se conoce de otros similares. En esta charla vamos a hacer un recorrido de los descubrimientos realizados durante la última década respecto al género imaginario: su espectro, su espectro abeliano, su relación con el género real, el grupo completo de automorfimos,...

Extensions of Fourier analysis arising from number theory

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In analytic and combinatorial number theory, many applications of Fourier analysis rely on the following idea: the averages of a function over certain linear patterns in an abelian group can be usefully analyzed by approximating the function using its dominating Fourier components. Since the 1990s, this idea has been considerably extended by the progress of a theory known as higher-order Fourier analysis. This theory grew out of seminal work of Gowers concerning a central result in combinatorial number theory, namely Szemerédi's theorem. The theory has also powered celebrated results such as asymptotic estimates for counts of various types of linear patterns in the primes, notably in work of Green and Tao. A key insight in higher-order Fourier analysis is that for many types of linear configurations, while the approximations by dominating Fourier components may not be helpful, it is still possible to carry out a very useful analysis by approximating the function by dominating components that are not defined using the circle group anymore, but rather using certain inherently non-commutative generalizations, such as nilmanifolds. Among the developments generated by this insight, there is the study of fascinating structures called nilspaces. These structures, which constitute a common generalization of abelian groups and nilmanifolds, have yielded further progress in the theory of higher-order Fourier analysis. I will provide an introduction to this theory and discuss some recent results in this approach involving nilspaces, based on joint work with Balázs Szegedy.

The 2875 lines within a general quintic hypersurface of $\mathbb{P}^4$

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There are exactly 2875 straight lines contained within a general quintic hypersurface of ¶^{4}. Algebraic Geometry will help us solve this classical problem of Enumerative Geometry. Mainly, well known tools of Intersection Theory will be used.

The key to the solution is the fact that the family of lines within a projective hypersurface turns out to be the zero locus of a section of the sheaf Sym$^d(\mathcal{S}^*)$ over the grassmannian $\mathbb{G}(1,4)$. Therefore, its class in the Chow ring is a Chern class - the top Chern class $c_6($Sym$^5(\mathcal{S}^*))$ - and its degree is shown to be exactly the number of lines in a general quintic hypersurface. Finally, the Excision Principle and some Schubert Calculus will unveil this value just from arithmetics and mere geometric intuition.

Bifurcations on a discontinuous Leslie-Grower model with harvesting and alternative food for predators and Holling II functional response

Christian Camilo Cortés García

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This paper proposes a mathematical model that describes the interaction of prey and predators, assuming logistic growth for both species, harvesting and alternative food for predators and functional response of the Holling II predator. When performing a qualitative analysis to determine conditions on the parameters that allow the possible extinction or preservation of prey and/or predators, a modification is made to the initial model considering that the consumption of prey by predators is restricted if the amount of prey is less than a critical value, whose dynamics is formulated by a planar Filippov system. The study of the discontinuous model is carried out by bifurcation analysis in relation to two parameters: harvesting of predators and critical value of prey.

Hyperbolicity and Chaos in Linear Dynamics: an overview on recent results

Emma D'Aniello

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We investigate in the linear setting hyperbolic notions and concepts from topological dynamics originally defined in a non-linear framework. We focus in particular on composition operators on arbitrary measure spaces.

Vertex Algebras and Geometric Structures

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Vertex algebras, introduced by Borcherds to prove the Monstruous Moonshine Conjecture, play an important role in many areas of mathematics, such as representation theory and Conformal Field Theory. They have had a strong impact in geometry recently, first by the construction of the Chiral De Rham Complex by Malikov-Schechtmann-Vaintrob, and, more recently, by the construction of superconformal structures on this complex by Heluani-Zabzine, among others.

The Chiral De Rham Complex can be constructed over Courant algebroids, the objects of study in generalized geometry. My research over the last years has revolved around how geometric structures from generalized geometry appear in the vertex algebra quantization of Courant algebroids. We have obtained results for Courant algebroids over a point, where generalized geometry connects with the representation theory of quadratic Lie algebras. These can be found on a joint work with Luis Álvarez-Cónsul and Mario Garcia-Fernandez in arxiv:2012.01851.

More technically, I will present a new method to obtain embeddings of the N=2 superconformal vertex algebra into superaffine vertex algebras, the ones associated to quadratic Lie algebras. The new input for this construction is a solution of the "Killing spinor equations" on the quadratic Lie algebra. These equations can be regarded as purely algebraic conditions, but, in fact, come from geometry and physics, specifically from the approach to special holonomy based on generalized geometry on Courant algebroids. As an application, I will give geometric examples.

Harmonic mappings and the Schwarzian derivative

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According to a classical theorem of Nehari an analytic function in the unit disk is univalent if its Schwarzian derivative

$$Sf = \left(\frac{f''}{f'}\right)' - \frac{1}{2}\left(\frac{f''}{f'}\right)^2$$

is small, in certain sense. This theorem has been generalized to functions defined on quasidisks by Ahlfors, on finitely connected domains (all of whose boundary components are either points or quasicircles) by Osgood and, finally, on uniform domains by Martio-Sarvas. Moreover, criteria for the existence of a quasiconformal extension have been proved on these types of domains, respectively, by Ahlfors-Weill, Ahlfors, Osgood and Astala-Heinonen.

In this talk we will generalize these theorems to harmonic mappings. Moreover, we will give two explicit quasiconformal extensions for harmonic mappings on the unit disk that generalize the classical extension of Ahlfors and Weill for analytic functions.

References

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The free Banach lattice over a Banach space

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Despite its short life, the free Banach lattice over a Banach space has proven to be a useful tool in Banach lattice theory. It can be conceived as a free object in the category of Banach lattices along with lattice homomorphisms, adapted to preserve the Banach space properties of the generating set. The existence of this object can be shown by means of an explicit construction, which allows to establish some of its basic properties.

For instance, one of the main features of the free Banach lattice over a Banach space is that it encodes some of the properties of the generating Banach space in terms of lattice properties. A similar situation arises if we look at a special class of lattice homomorphisms between free Banach lattices: the ones induced by bounded linear operators between the underlying Banach spaces. On the other hand, the idea behind the free Banach lattice can be used to extend this construction to the context of Banach lattices with additional properties, such as p-convexity ($1 \leq p \leq \infty$).

The last part of the talk will be devoted to introduce this setting, while presenting some results concerning the structure of this family of spaces, their relation with biduality and some properties of complementation.

Finite rank perturbations of normal operators: invariant subspaces and decomposability

F. Javier González-Doña

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In this talk, we consider compact perturbations of diagonalizable normal operators acting boundedly on infinite-dimensional, separable, complex Hilbert spaces. In particular, we will present results regarding the existence of non-trivial closed hyperinvariant subspaces for for finite-rank perturbations of diagonalizable, normal operators extending previous results of Foias, Jung, Ko and Pearcy [2,3,4] and Fang and Xia [1]. We will address the main techniques involved in the proof, based on Local Spectral Theory.

Finally, we will introduce the concept of spectral decomposability of an operator and show that a large class of such operators are decomposable, having, consequently, a rich lattice of invariant subspaces.

This is part of joint works with Eva A. Gallardo-Gutiérrez [5,6,7].

References

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Factorizar enteros con informacion extra

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Dado un entero n=pq producto de dos primos queremos saber si existe un entero m coprimo con n tal que la factorizacion de m permite factorizar n en tiempo polinomico. En 2006 Pailler y Villar conjeturaron que tal m no existe. Con Dieulefait probamos que esta conjetura es falsa. Del trabajo, continuamos para de demostrar finalmente que factorizar un entero n=pq es equivalente a contar puntos en curvas elipticas. Muy recientemente, junto con Pomykala, extendemos el algoritmo a cualquier n libre de cuadrados.

Variational principles in physics and their geometry. An introduction to the Herglotz principle

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A trajectory c will be followed by the system if and only if c is a critical point of the action among all trajectories with the same endpoints as c." If by the action we mean the integral of a Lagrangian function L along c, this is Hamilton's least action principle which can be used to describe most physical theories (geometric optics, classical mechanics, electrodynamics, general relativity, etc.), and has applications in other fields (for example, solving optimal control problems and finding geodesics in Riemannian and Finsler geometry). Its solutions have a nice geometric characterization: they are integral curves of a Hamiltonian vector field on a symplectic manifold. Part of my current research deals with a generalization of this principle: the so-called Herglotz principle. Here the Lagrangian not only depends on the positions and velocities, but also on the action. Hence, the action is no longer the integral of the Lagrangian, but it is the solution of an ODE. This principle allows us to model new problems, such as some dissipative systems in mechanics (where energy is lost), thermodynamics, and some modified optimal control systems. This principle is also related to Hamiltonian systems, but switching symplectic by contact geometry. In this talk we will compare both principles, their applicability and the geometric properties of their solutions.

If time permits, we will discuss a possible extension of the Herglotz principle to fields.

The distance to the border of a random tree

Victor Maciá

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In this talk we will give an asymptotic formula for the probability that a random tree has distance to the border bigger or equal than k, as the number of nodes escapes to infinity.

Composition operators and weighted shifts: how are they related?

Martina Maiuriello

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Chaotic and hyperbolic properties for composition operators have been widely analyzed in the last decades. In the vast literature on the subject, it emerges how some dynamical properties are characterized for weighted shifts even before the theory on the property itself is clearly understood and revealed. It turns out that weighted shifts are a good starting point for the analysis of composition operators, leading to the following natural question: when and how is it possible to deduce the dynamical properties of a composition operator from those of an appropriately associated weighted shift? This talk attacks the above question, developing a technique that takes known characterizations for weighted shifts and, in a certain way, lifts them up to the large class of dissipative composition operators of bounded distortion, called *shift-like operators*.

Convex integration in fluid mechanics

Javier Peñafiel Tomas

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Convex integration is a technique developed by Nash to construct isometric embeddings of Riemannian manifolds. In the last decade, convex integration has gained much popularity because it has been successfully applied to the Euler equations by De Lellis and Székelyhidi.

In this talk, we will introduce this technique and show how to use it to construct steady weak solutions to the Euler equations. We will focus on the stationary case but emphasize the differences with the time-dependent case. We will also give an overview of the problems solved using this construction, namely Onsager’s conjecture about anomalous dissipation, as well as some open questions.

Zero location of extremal Sobolev polynomials

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In this talk, we work on the location of zeros of extremal Sobolev polynomials. The asymptotic distribution of zeros is established on general conditions. Under some order restriction in the discrete part, we prove that the n-th extremal polynomial has at least $n - \mathbf{d}^*$ zeros on the convex hull of the support of the measure, where $\mathbf{d}^*$ denotes the number of terms in the discrete part. This is a joint work with Héctor Pijeira Cabrera and Abel Díaz González.

Zero location of extremal Sobolev polynomials

Alejandro Quintero Roba

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The One-Dimensional Krein Laplacian Self-Adjoint Operator and Sequences of Krein-Sobolev Orthogonal Polynomials

Distribución de los primos y aproximación en espacios de tipo Dirichlet

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Estudiamos regiones sin ceros para la función ζ de Riemann, en relación a un problema de aproximación en el espacio de tipo Dirichlet D_{-2}, del cual se sabe que es equivalente a la hipótesis de Riemann, a partir del trabajo de Báez-Duarte. Demostramos, de hecho, que problemas análogos de aproximación para los espacios de tipo Dirichlet D_α con $\alpha \in (-3, -2)$ proporcionan condiciones para que el semiplano $\{s \in \mathbb{C} : \Re(s) > -\frac{\alpha+1}{2}\}$ sea asimismo libre de ceros de ζ. Más aún, extendemos estos resultados a una gran familia de espacios de tipo l^p con pesos, de funciones analíticas. Como aplicación, en el caso límite $p = 1$ y $\alpha = -2$, damos una nueva prueba del Teorema del Número Primo

An introduction to higher mathematics

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As working mathematicians, we tend to model every mathematical idea (such as a function, a group or a space) in terms of a set with a certain structure, while we think of category theory as a tool for doing metamathematics. However, the idea that small categories (such as action groupoids) can themselves have interesting structure and represent natural mathematical ideas in a richer way than sets do has attracted attention over the last years, motivated by areas such as the theory of moduli spaces in geometry or the study of higher dimensional sigma-models in theoretical physics. In this talk I will present some basic definitions of this approach to mathematics and I will try to motivate it by showing how categories can be used to give simple smooth models for certain complicated topological spaces, such as the String group, that do not admit a classical structure of manifold.