Topological Methods in Population Dynamics

Héctor Barge Yáñez
Universidad Complutense de Madrid

Let X be a locally compact metric space and let $E \subset X$ be a closed subset with nonempty boundary and interior. Let $\varphi : E \times \mathbb{R} \to E$ be a dissipative flow on E for which ∂E is invariant. Usually the space E is chosen as the nonnegative orthant of Euclidean space, because populations are constituted by a nonnegative number of elements. We say that the flow φ is uniformly persistent if there is a $\beta > 0$ such that

$$\liminf_{t \to \infty} d(\varphi(x,t), \partial E) \geq \beta, \quad \text{for all} \quad x \in int(E)$$

The notion of uniform persistence is an important feature in some dynamical systems used to model the time evolution of populations. This feature is equivalent to the existence of an asymptotically stable attractor that attracts all interior points of E. This notion reflects the idea that none of the species which make up an ecosystem becomes extinct over time and instead, all states evolve towards stable coexistence.

In this talk a classical result which states sufficient conditions for uniform persistence that can be simply verified by studying the flow near the boundary ∂E will be introduced. This result is known as the **Butler-Waltman-Freedman Theorem**. In addition we will study some features of uniformly persistent flows using topological techniques such as Borsuk's shape theory, Conley index theory or Morse theory. These techniques allow us to provide sufficient conditions for the preservation of uniform persistence by small perturbations of the flow φ or, in some cases, infer the existence of local attractors with circular shape contained in the global attractor.