Isolated initial singularities for the viscous Hamilton-Jacobi equation

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Here, we study the nonnegative solutions with a possible singularity at the point $(x,t) = (0,0)$ of the viscous Hamilton-Jacobi equation:

$$u_t - \Delta u + |\nabla u|^q = 0$$

in $Q_{\Omega,T} = \Omega \times (0,T)$, where $q > 1$, $T \in (0,\infty]$, and Ω is a smooth bounded domain in $\mathbb{R}^N$. We show that if $q \geq q^* = \frac{N+2}{N+1}$, then the singularity is removable. In the case, $1 < q < q^*$, we prove the existence and uniqueness of a very singular solution for the Dirichlet problem, and the Cauchy problem without assumption at the infinity. This leads to a complete description of singular solutions.