

# Hyperbolicity in the Gromov sense of the strong product of two graphs.

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## Abstract

If  $X$  is a geodesic metric space and  $x_1, x_2, x_3 \in X$ , a *geodesic triangle*  $T = \{x_1, x_2, x_3\}$  is the union of the three geodesics  $[x_1x_2]$ ,  $[x_2x_3]$  and  $[x_3x_1]$  in  $X$ . The space  $X$  is  $\delta$ -*hyperbolic* (in the Gromov sense) if any side of  $T$  is contained in a  $\delta$ -neighborhood of the union of the other two sides, for every geodesic triangle  $T$  in  $X$ . In this paper we characterize the strong products of two graphs  $G_1 \boxtimes G_2$  which are hyperbolic, in terms of  $G_1$  and  $G_2$ : the strong product graph  $G_1 \boxtimes G_2$  is hyperbolic if and only if one of the factors is hyperbolic and the other one is bounded. We also prove some sharp relations between  $\delta(G_1 \boxtimes G_2)$ ,  $\delta(G_1)$ ,  $\delta(G_2)$  and the diameters of  $G_1$  and  $G_2$ .