

Extensions of topological abelian groups, quasi-characters and three-space problems

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An extension in the category of topological abelian groups is a short exact sequence $0 \rightarrow H \rightarrow X \rightarrow G \rightarrow 0$, where all maps are assumed to be continuous and open onto their images. The extension splits if it is equivalent to the trivial one $0 \rightarrow H \rightarrow H \times G \rightarrow G \rightarrow 0$ in the usual homological sense; this equivalence must, of course, be witnessed by a topological isomorphism.

We study some permanence properties and notable subclasses of the class $S_{TG}(\mathbb{T})$ of topological abelian groups G for which every extension of the form $0 \rightarrow \mathbb{T} \rightarrow X \rightarrow G \rightarrow 0$ splits. Among other techniques we use the connection between these extensions and quasi-characters on G , as well as three-space problems for topological groups.

This line of work, which was previously explored by F. Cabello and others, is inspired by the well established theory of twisted sums of F -spaces and the concept of a \mathcal{K} -space, which play a central role in the negative solution of the three-space problem for local convexity.

(Joint work with Hugo J. Bello and M. Jesús Chasco)