Lie–Jordan Banach algebras and Quantum Reduction

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A unified approach to classical and quantum mechanics is presented and is based on the use of Lie–Jordan Banach algebras. It leads to insights into the nature of the quantization procedure, the classical limit of quantum mechanics and the role of symmetries and constraints.
Lie–Jordan Banach algebras are the algebras of observables of quantum mechanics and completely characterize the self-adjoint part of $\mathbb{C}$–algebras among normed Jordan algebras. Taking as a motivation the classical theory of reduction of Poisson algebras, a theory of reduction of Lie–Jordan Banach algebras is developed and compared with the T-procedure reduction of $\mathbb{C}$–algebras. The space of states of the reduced Lie–Jordan Banach algebras is described in terms of equivalence classes of extensions to the full algebra and their GNS representations are characterized in the same way.