Variable exponent Lebesgue spaces and modulars

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Variable exponent Lebesgue spaces $L^{p(\cdot)}(\Omega, \mu)$ are a natural generalization of the classical Lebesgue spaces and belong to the general class of modular spaces. They appeared in the mathematical literature for the very first time in the early 30's in articles by Orlicz and Nakano. In the last two decades, there has been a strong renewed interest in the field, motivated for their applications to Harmonic Analysis, Fluid Mechanics and Physics.[1]

In the Variable exponent Lebesgue spaces, instead of having a fix exponent p, as we do in the classical Lebesgue Spaces, we considerer a *exponent function* $p(\cdot)$, which takes values in the interval $[1, \infty]$.

It comes out that many properties and results of the classical Lebesgue spaces verify a natural generalization in this more general context, however, there are others which may fail dramatically. This abrupt difference in behavior lies in the fact that Variable exponent Lebesgue spaces are *not symmetric* (rearrangement invariant).

In this work, after giving a short introduction in the theory of Modular Spaces, which would provide us the framework to introduce the Variable exponent Lebesgue Spaces, their basic properties will be studied. Finally, in the last section, we will focus on the lattice and ℓ_q- structure of this spaces, where recent discoveries have been made.

At this stage, among many others results, we will prove that, unlike the case of the classical Lebesgue Spaces, there might be *unbounded* orthogonal ℓ_q-projections in this spaces.

[1] M. Ružička, Electrorheological Fluids: Modeling and Mathematical Theory, Lecture Notes in Math., vol. 1748, Springer, 2000.