On Fourier Series of Jacobi-Sobolev Orthogonal Polynomials

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Let μ be the Jacobi measure on the interval $[-1,1]$ and introduce the discrete Sobolev-type inner product

$$\langle f,g\rangle = \int f(x)g(x)d\mu(x) + \sum_{k=1}^{K}\sum_{i=0}^{N_k} M_{k,i} f^{(i)}(b_k)g^{(i)}(b_k),$$

where the mass points b_k belong to $\mathbb{R}\backslash supp(\mu)$ and $M_{k,i} \geq 0$ with $M_{k,N_k} \neq 0$. The pointwise convergence of the Fourier Series associated with this inner product is studied. In such a way we generalize the results given in [1]. The technique used here requires estimates for the Jacobi-Sobolev orthogonal polynomials at the mass points. To this end, we obtain the order of the ratio $\frac{q_n^{(k)}(x)}{q_n(x)}$, when $k < n$ and $x \in \mathbb{C}\backslash[-1,1]$. Here $(q_n(x))_{n=0}^{\infty}$ is the sequence of orthonormal polynomials for a measure μ in the Szegő class.

[1] F. Marcellán, B. P. Osilenker, I. A. Rocha, On Fourier Series of Jacobi-Sobolev Orthogonal Polynomials, J. Inequal. Appl. **7**(5) (2002), 673-699.