Some variational principles and applications to the Geometry of Banach spaces

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In 1972, Ivar Ekeland obtained a variational principle (V.P.) which has been a key step to obtain some geometric properties of Banach spaces and several fixed point theorems. Throughout this work, we will review and study this result and other variational principles like the Stegall's V.P., the Deville, Godefroy and Zizler's V.P., the Borwein-Preiss V.P. and some of their many applications. We will also study several parametric variational principles (P.V.P.) by P. Georgiev, L. Veselý, R. Deville, A. Procházka, ... From these results alternative proofs on Nash equilibriums, a parametric version of the Karush-Kuhn-Tucker theorem, several properties of convex sets and convex functions, etc... are obtained. Finally, we will briefly introduce the vector-valued variational principles.