Alliance Polynomial of a Graph.

Yadira Torres Nuñez

Abstract

The concepts of alliances in graphs were introduced by P. Kristiansen, S. M. Hedetniemi and S. T. Hedetniemi in 2004. Colloquially speaking, a defensive k-alliance in a graph G is a set S of vertices of G such that every vertex in S has at least k more neighbors in S than it has outside of S. We say that S is an exact defensive k-alliance in G, if S is defensive k-alliance but is not defensive $(k+1)$-alliance.

The alliance polynomial of a graph G with order n and maximum degree Δ is the polynomial $A(G;x) = \sum_{k=-\Delta}^{\Delta} A_k(G)\, x^{n+k}$, where $A_k(G)$ is the number of exact defensive k-alliance in G. We obtain some properties of $A(G;x)$ and its coefficients. In particular, we prove that the path, cycle, complete and start graphs are characterized by their alliance polynomials.

Besides, we study the alliance polynomial for cubic graphs, which verify unimodality. Computationally we obtain alliance polynomials for cubic graphs of small order, which verify uniqueness.