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WHEN IS THE TYPICAL OPERATOR NORM ATTAINING?

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ABSTRACT. We will present the relationship between the residuality of the set of norm attaining functionals on a Banach space and the residuality and the denseness of the set of norm attaining operators between Banach spaces.

On the one hand, we show that if C is a bounded subset of a Banach space X which admit an LUR renorming satisfying that, for every Banach space Y, the operators T from X to Y for which the supremum of $\|Tx\|$ with $x \in C$ is attained are dense, then the G_δ set of those functionals which strongly exposes C is dense in X^*. This extends previous results by Bourgain and Lau. The particular case in which C is the unit ball of X, in which we get that the norm of X^* is Fréchet differentiable at a dense subset, improves a result by Lindenstrauss and we even present an example showing that Lindenstrauss' result was not optimal.

In the reverse direction, results for the density of the G_δ set of absolutely strongly exposing operators from X to Y by requiring that the set of strongly exposing functionals on X is dense and conditions on Y or Y^* involving RNP and discreteness on the set of strongly exposed points of Y or Y^* are shown. These results include examples in which even the denseness of norm attaining operators was unknown.

These results find important applications to the classical theory of norm-attaining operators, to the theory of norm-attaining bilinear forms, to the geometry of the preduals of spaces of Lipschitz functions, and to the theory of strongly norm-attaining Lipschitz maps.

We will first give an overview on results on norm attaining operators to put the new results into perspective.