Libro de Abstracts

XVI Workshop de Jóvenes Investigadores

Organizado por:

Facultad de Ciencias Matemáticas

Universidad Complutense de Madrid

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Workshop de Jóvenes Investigadores

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High-energy eigenfunctions and inverse localization

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A well-known link between solutions to the Helmholtz equation $\Delta h + h = 0$ and eigenfunctions of the Laplacian on a closed Riemannian manifold is that the local behaviour of a sequence of high-energy eigenfunctions (say,$\lambda \to \infty$) defines a bounded Helmholtz solution, after a suitable rescaling. Conversely, every solution to Helmholtz can be locally realized by an approximate eigenfunction of any large enough energy, on scales determined by this energy.

A powerful refinement of the latter fact is what we call the inverse localization principle: if, roughly speaking, the degeneracy of the high-energy eigenvalues is large enough, one can replace the quasimodes by bona fide eigenfunctions.

In this joint work with A. Enciso and D. Peralta-Salas, we consider the question of when the Laplace eigenfunctions on an arbitrary flat torus are flexible enough to approximate, over the natural length scale of order 1/ an arbitrary solution of the Helmholtz (i.e. when it satisfy the inverse localization property).

A walk through eigenfunctions' realm

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In this talk we will describe to a general mathematical audience a realm of mathematics that lies at the intersection of algebra, geometry and analysis (with connections to many others). The emphasis will be on different open problems that we hope the audience will find enticing.

Existence of vector spaces of linear discontinuous functions in an abstract setting

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We say that a function $f\colon \mathbb{R} \to \mathbb{R}$ is additive if it satisfies Cauchy's functional equation $f(x+y) = f(x) + f(y)$ for every $x, y \in \mathbb{R}$. It is well-known that every additive function $f\colon \mathbb{R} \to \mathbb{R}$ is $\mathbb{Q}$-linear, i.e., f is additive and $f(qx) = qf(x)$ for every $q \in \mathbb{Q}$ and $x \in \mathbb{R}$. Moreover, if $f\colon \mathbb{R} \to \mathbb{R}$ is an additive continuous function, then f is of the form $f(x) = ax$ for some $a \in \mathbb{R}$ (in particular, f is $\mathbb{R}$-linear, that is, f is additive and $f(rx) = rf(x)$ for every $r, x \in \mathbb{R}$). However, there are additive functions that are discontinuous (in fact, these type of functions are discontinuous everywhere).

García-Pacheco et.al. studied the existence of infinite dimensional vector spaces of functions of the form $f\colon V \to \mathbb{R}$ that are discontinuous and additive, where V is a non-discrete real topological vector space. In this talk, we will extend their results and provide new ones in a more abstract setting by considering families of functions of the form $f\colon X \to \mathbb{K}$, where $\mathbb{K}$ is a field endowed with a non-trivial absolute value and X is either $\mathbb{K}$ or a non-discrete topological vector space defined over $\mathbb{K}$.

Joint work with Fernández-Sánchez, Seoane-Sepúlveda and Trutschnig.

Subharmonics in planar Hamiltonian systems: an index theory approach

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In this talk, we present a recap of the index theory for linear planar Hamiltonian systems with periodic coefficients providing the relationship between the stability of the system, the Conley-Zehnder index, the winding number of solutions and their respective asymptotic indices. Although most of these relationships are wellknown, we take a slightly different a priori classification in order to give a more natural definition of the Conley -Zehnder index for resonant cases from the point of view of the rotational behavior of solutions. This seems to be more appropriate to the analysis of the existence and multiplicity of subharmonics in nonlinear planar Hamiltonian systems via the Poincaré-Birkhoff theorem, as will be discussed in the second part of the talk. This is a joint work with Alberto Boscaggin from the University of Turin.

Schur multipliers and noncommutative Fourier multipliers

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Schur multipliers form a class of linear maps on matrix algebras and operator algebras with profound connections in functional analysis, operator algebras, geometric group theory and harmonic analysis. In this talk we shall introduce this objects and present a rather simple criterium for its boundedness on their natural L_p spaces, the Schatten-p classes. Moreover, we will introduce the noncommutative Fourier multipliers and we will explore some applications of the result in the theory of noncommutative Fourier multipliers on Lie groups.

Electrostatic model for Sobolev ortogonal polynomials

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We study the sequence of orthogonal polynomials with respect to a discrete classical Sobolev inner product. We provide a formula that relates the Sobolev polynomials to the classical orthogonal polynomials. We find the corresponding ladder operators and a second-order differential equation with polynomial coefficients. We establish an electrostatic model for their zeros.

The reach of isometric embeddings into Wasserstein spaces

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First we recall the definition of the reach of a subset of a metric space, in the sense of Federer. Let (X, d) be a metric space and $A \subset X$ a subset. We define the set of points having a unique metric projection in A as

$$(A) = \{x \in X : \text{there exists an unique } a \text{ such that } d(x, A) = d(x, a)\}.$$

For $a \in A$, we define the *reach* of A at a, denoted as (a, A), as

$$(a, A) = \sup\{r \geq 0 : B_r(a) \subset (A)\}.$$

Finally, we define the *global reach* of the subset A as

$$(A) = \inf_{a \in A} (a, A).$$

The intuitive idea is that, $(A) = 0$ if and only if we do not have a neighbourhood of A admitting an unique metric projection into A, even locally around a point. This can be understood as a measure of how $\varphi(X)$ is wrapped inside its ambient space.

Now, given a geodesic metric space $(X,)$, one can equip the space of probability measures supported on X with a distance induced by the solutions to an optimal transport problem. Usually the cost comes from taking the $p-$power of the distance function, the so called $p-$*Wasserstein spaces*. Then, the distance between two probability measures will be the cost of transporting one to the other (assuming it is transported in an optimal way).

We consider the isometric embedding $x \mapsto \delta_x$ to study the reach of X as a subset of the Wasserstein space $W_p(X)$, obtaining results such as:

- If X is a geodesic space where two points x, y are joint by two different minimizing geodesics, then $(X \subset W_p(X))$ is 0. This is the case for non contractible manifolds.

- If X verifies certain convexity assumptions, then $(X \subset W_p(X))$ is ∞.

We also prove similar results in other Wasserstein-type spaces.

Polynomial images of closed balls

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In this talk (based on the article: J.F. Fernando, C. Ueno: On polynomial images of a closed ball. J. Math. Soc. Japan 75 (2023), no. 2, 679-733) we approach the problem of determining which (compact) semialgebraic subsets of $\mathbb{R}^n$ are images under polynomial maps $f : \mathbb{R}^m \to \mathbb{R}^n$ of the closed unit ball $\mathcal{B}_m$ centered at the origin of some Euclidean space $\mathbb{R}^m$ and that of estimating (when possible) which is the smallest m with this property. Contrary to what happens with the images of $\mathbb{R}^m$ under polynomial maps, it is quite straightforward to provide basic examples of semialgebraic sets that are polynomial images of the closed unit ball. For instance, simplices, cylinders, hypercubes, elliptic, parabolic or hyperbolic segments (of dimension n) are polynomial images of the closed unit ball in $\mathbb{R}^n$.

The previous examples (and other basic ones proposed in the article) provide a large family of 'n-bricks' and we find necessary and sufficient conditions to guarantee that a finite union of 'n-bricks' is again a polynomial image of the closed unit ball either of dimension n or $n + 1$. In this direction, we prove: *A finite union $\mathcal{S}$ of n-dimensional convex polyhedra is the image of the n-dimensional closed unit ball $\mathcal{B}_n$ if and only if $\mathcal{S}$ is connected by analytic paths*.

The previous result can be generalized using the 'n-bricks' mentioned before and we show: *If $\mathcal{S}_1, \dots, \mathcal{S}_\ell \subset \mathbb{R}^n$ are 'n-bricks', the union $\mathcal{S} := \bigcup_{i=1}^{\ell} \mathcal{S}_i$ is the image of the closed unit ball $\mathcal{B}_{n+1}$ of $\mathbb{R}^{n+1}$ under a polynomial map $f : \mathbb{R}^{n+1} \to \mathbb{R}^n$ if and only if $\mathcal{S}$ is connected by analytic paths*.

Species interactions reproduce abundance correlation patterns in microbial communities

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During the last decades macroecology has identified broad-scale patterns of abundances and diversity of microbial communities and put forward some potential explanations for them. However, these advances are not paralleled by a full understanding of the dynamical processes behind them. In particular, abundance fluctuations over metagenomic samples are found to be correlated, but reproducing populations through appropriate population models remains still an open task. The present paper tackles this problem and points to species interactions as a necessary mechanism to account for them. Specifically, we discuss several possibilities to include interactions in population models and recognize Lotka-Volterra constants as successful ansatz. We design a Bayesian inference algorithm to obtain sets of interaction constants able to reproduce the experimental correlation distributions much better than the state-of-the-art attempts. Importantly, the model still reproduces single-species, experimental, macroecological patterns previously detected in the literature, concerning the abundance fluctuations across both species and communities. Endorsed by the agreement with the observed phenomenology, our analysis provides insights on the properties of microbial interactions, and suggests their sparsity as a necessary feature to balance the emergence of different patterns.

Poincaré-Hopf Theorem, Borsuk-Ulam Theorem and Dynamical Systems

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We propose to study some relationships between the Brouwer degree of a vector field and the dynamics of the induced flow. We are interested, in particular, in the dynamic and topological properties of the isolated invariant sets and their unstable manifolds. We also study analogous relationships for the index of a vector field and in this way we obtain new forms of the Poincaré-Hopf theorem. We also obtain some consequences regarding the antipodal theorems of Borsuk-Ulam and Hirsch. As an application, we calculate the Brouwer degree and the vector field index in some topologically relevant situations, we obtain criteria for the detection of connecting orbits in the attractor- repeller decompositions of the isolated invariant sets and we calculate the Brouwer degree of the vector field of the Lorenz equations in isolating blocks of the strange set. These results have been obtained in collaboration with Héctor Barge.

This derivation likes to leap

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Given a k-algebra, with k a commutative ring, determining whether a derivation is infinite-integrable is not a simple problema in general. In this talk, we will give the main concepts and results of the theory of Hasse-Schmidt derivations, integrability and of course, leaps.

Contractions of Lie algebras and applications to ODEs systems

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Contractions of Lie algebras were introduced in Physics to formally describe the idea that some system is obtained from another by a kind of limiting process as, for example, the Galilean algebra is a speed-space contraction of the Poincaré algebra. In this talk we will present the novel notion of contraction of Lie systems —these are, ODEs systems admitting a (nonlinear) superposition rule—, showing how the so-called Lie—Hamilton systems on the plane possessing a three-dimensional Vessiot—Guldberg algebra are related by this notion of contraction. These relations will be obtained using the classification of contractions of three-dimensional real Lie algebras, which will be deduced through a new cohomological approach using the de Rham and adjoint cohomology of Lie algebras.

Partition identities via commutative algebra

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A partition of a positive integer n is a sequence $\lambda : (\lambda_1 \geq \cdots \geq \lambda_l)$ such that $\lambda_1 + \cdots + \lambda_l = n$. The integer λ_i are called *the parts of* λ. A *partition identity* is an equality between the number of partitions of n satisfying some property A and the number of partitions of n satisfying another property B, which is true for any positive integer n. These identities play an important role in many areas such as number theory, combinatorics, Lie theory, particle physics and statistical mechanics. An important example of these identities is called the *first Rogers-Ramanujan identity*:

Theorem. *The number of partitions of a positive integer* n *with no equal or consecutive parts is equal to the number of partitions of* n *into parts* $\equiv 1$ *or* $4(mod.5)$.

Using the relation between the *Hilbert-Poincaré series* of the graded algebras associated to the *arc spaces*, we add a new member to this famous identity.

Geometry of polynomial spaces and applications

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In this talk we shall focus on studying the geometry of the normed space of trinomials on two different scenarios and their applications. In particular, the norms are defined in $\mathbb{R}^3$ as

$$\|(a,b,c)\|_{m,n} := \sup\{|ax^m + bx^n + c| : x \in [-1,1]\},$$
$$(a,b,c)_{m,n} := \sup\left\{|ax^m + bx^{m-n}y^n + cy^m| : (x,y) \in [-1,1]^2\right\},$$

for $m,n \in \mathbb{N}$ with $m > n$. We provide an explicit formula to calculate the norm, a full description of the extreme points of the corresponding unit balls and a parametrization and a plot of their unit spheres depending on the parity of both m and n. As an application, we explain how this work can be used to obtain sharp Bernstein and Markov inequalities, or Bohnenblust-Hille inequalities, using the *Krein-Milman approach*.

Part of the content of this lecture is the result of a joint work with Profs. Gustavo A. Muñoz Fernández (UCM), Juan B. Seoane Sepúlveda (UCM), and Mingu Jung (Korea Institute for Advanced Study, Seoul, South Korea).

On nuclear operators and when they attain their norms

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In this talk, we introduce and study a concept of norm-attainment in the space of nuclear operators $\mathcal{N}(X, Y)$ and in the projective tensor product space $X \otimes_\pi Y$. We give the first positive and negative results where both norm-attainment hold. We also present the problem on whether the class of elements which attain their norms in $\mathcal{N}(X, Y)$ and in $X \otimes_\pi Y$ is dense or not. We prove that, for both concepts, the density of norm-attaining elements holds for a large class of Banach spaces, which, in particular, covers all the classical Banach spaces. Despite of that, we present Banach spaces X and Y such that the class of elements in $X \otimes_\pi Y$ which attain their projective norms is not dense. During the whole talk, we will be trying to discuss some relations and applications of our work to the classical theory of norm-attaining operators.

This is a joint work with Mingu Jung, Óscar Roldán, and Abraham Rueda Zoca.

New reverse Hölder-type inequalities and applications

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Integral inequalities are a fundamental tool in mathematics and have countless applications in various fields. They allow us to establish bounds on integrals and compare the values of different integrals, and are an essential part of many mathematical theories and techniques. In recent years there has been a growing interest in the study of many classical inequalities applied to integral operators associated with different types of fractional derivatives, since these fractional integral inequalities have numerous applications in the theory of differential equations, applied mathematics, physics, engineering, and finance. In this talk, we establish several Hölder-type inequalities using Jensen-type and Young-type inequalities as key tools. Particularly noteworthy is a reverse Hölder inequality with the Specht's ratio. Additionally, we obtain a reverse Young-type inequality and we apply these results to the fractional context, both globally and locally.