PROPUESTA DE TRABAJO DE FIN DE MÁSTER
MÁSTER EN MATEMÁTICAS AVANZADAS

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Curso:

Título: Convergence and approximation properties of neural networks through Functional Analysis

Resumen:

Neural networks and Functional Analysis share a deep connection in the realm of mathematics and computational science. Functional Analysis, as a branch of mathematics, provides a theoretical foundation for understanding the properties and behaviour of functions, including those used in neural networks. In the context of neural networks, Functional Analysis is often applied to study the convergence and approximation properties of network architectures and the mathematical underpinnings of optimization algorithms used in training. The study of function spaces, operators, and normed vector spaces in Functional Analysis provides essential tools for analyzing the convergence of neural network models, which is crucial for ensuring their reliability and performance. Moreover, Functional Analysis helps to establish the mathematical rigor behind the expressive power of neural networks, shedding light on their ability to approximate complex functions and thus underpinning their effectiveness in various real-world applications. The interplay between neural networks and Functional Analysis highlights the importance of mathematical theory in advancing the field of deep learning and artificial intelligence. Our master's thesis proposal aims to investigate the convergence and approximation properties of neural networks by leveraging the powerful tools and concepts from Functional Analysis. By reviewing existing literature in the field, the student must synthesize and analyse the current state of knowledge surrounding the convergence behaviour of neural network architectures and their ability to approximate complex functions. In addition, the student must show that he is able of understanding the papers and identifying relevant open problems of the field. To perform the literature review, the student must 1 take a look at some of the main journals of Mathematical Analysis (at least "Journal of Differential Equation", "SIAM: Journal on Mathematical Analysis" and "Constructive Approximation") and study recent publications and the references therein. Also, the student must summarize the papers [GKNV22], where a framework of functional spaces for neural networks is presented, and [KPRS22], where the interaction of neural networks and PDEs is shown.

References

[GKNV22] Rémi Gribonval, Gitta Kutyniok, Morten Nielsen, and Felix Voigtlaender. Approximation spaces of deep neural networks. Constructive approximation, 55(1):259–367, 2022.
[KPRS22] Gitta Kutyniok, Philipp Petersen, Mones Raslan, and Reinhold Schneider. A theoretical analysis of deep neural networks and parametric pdes. Constructive Approximation, 55(1):73–125, 2022.