Book of Abstracts

XVIII Workshop de Jóvenes Investigadores

The talks will be held in the room Miguel de Guzmán at the Faculty of Mathematical Sciences of the Universidad Complutense de Madrid

September 23-25, 2024

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Abstracts

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Scattering Transform on non abelian finite groups

Scattering Networks were initially designed to elucidate the behavior of early layers in Convolutional Neural Networks (CNNs) over Euclidean spaces and are grounded in wavelets. In this work, we introduce a scattering transform on an arbitrary finite group (not necessarily abelian) within the context of group-equivariant convolutional neural networks (G-CNNs). We present wavelets on finite groups and analyze their similarity to classical wavelets. We demonstrate that any feature generated by such a network is stable under the group's action and Lipschitz continuous. Furthermore, we provide examples illustrating the application of the scattering transform to data with domains involving nonabelian groups. Joint work with Davive Barbieri and Eugenio Hernández.

Ana Bravo

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Order functions and applications to resolution of singularities

When working with algebraic varieties that are embedded in a smooth ambient space over a field of characterisitc zero, the usual order function plays a crucial role in the definition of the functions used in constructive resolution of singularities. To treat the case of abstract algebraic varieties, this approach forces us to choose local (étale) embeddings, and as a consequence, some work has to be done in order to prove that the functions obtained in this way do not depend on the many possible choices. In this talk we will focus on order functions and their properties in the (wider) context of local equidimensional excellent rings. We will see how they can help us to recover information about resolution functions using natural intrinsic methods. This is joint work with A. Benito, S. Encinas and J. Guillán-Rial.

(1) A. Benito, A. Bravo and S. Encinas, *The Asymptotic Samuel function and applications to resolution.* En preparación.

(2) A. Bravo, S. Encinas and J. Gillán-Rial, *On some properties of the Asymptotic Samuel function.* Preprint: arXiv2307.11489v1.

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Metric differentiability of Lipschitz mappings in metric measure spaces

We follow the spirit of the classical result of Rademacher in order to study regularity of Lipschitz mappings $f : X \to Y$, where X is a metric measure space and Y a metric space. To this end, we consider Cheeger's idea of differentiation using an atlas of charts in X, as well as the notion of metric differentiability introduced by Kirchheim. We prove that a Rademacher result follows in this setting if and only if X admits a decomposition into rectifiable charts.

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A Whole New Nonlocal World

Can you imagine if we could have a $\frac{3}{4}$ derivative?-

-What? No, why would I need to do that?

Well, as we all know, the use of mathematics has been quite effective in describing natural phenomena. Popularly through the use of (partial) differential equations to model systems in physics, biology or economics for example. The function that describes the system is 'hidden' in these differential equations which stablish a relation between a function and its derivatives (related to how the function changes). But if we stretch something, for example this wooden beam.... (crack!) a fracture appears! Thus, sometimes singularity phenomena may arise, and that implies functions with discontinuities which do not fit very well with these classical models.

There is something that can tackle this. A new fantastic point of view! What is nonlocal?

We will try to understand that. Basically, we will consider a relaxed notion of gradient, typically made of an integral of a difference quotient. As a consequence, less regularity is needed and long range interactions can be taken into account (nonlocal: points at a finite distance may exert an interaction upon each other).

This means we have new horizons to pursue! In particular, we will need to obtain several tools, to the extent possible, similar to those of the classical case, so that we can study these new models. Fortunately, we have already been able to obtain quite a few, where a key ingredient has been a nonlocal version of the fundamental theorem of calculus.

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A priori Estimates and Harnack Inequalities for Fractional Porous Medium type Equation

We study the positivity and regularity of a suitable class of nonnegative solutions to the fractional porous medium equations $u_t + (-\Delta)^s u^m = 0$ in $(0, \infty) \times \Omega$ for $m > 1$, $s \in (0, 1)$ and with homogeneous Dirichlet boundary data, and initial data $u(0, x) = u_0(x) \geq 0$ for $x \in \Omega$.

The main results are quantitative lower and upper bounds for solutions which holds for all positive times $t > 0$. As a consequence of these a priori estimates, we find a global Harnack principle of the form $C_1(t, u_0)d^{s/m}(x) \leq u(t, x) \leq C_2(t)d^{s/m}(x)$ for any $(t, x) \in (0, \infty) \times \Omega$, where d is the distance to the boundary of the domain Ω.

After this, we study the regularity of solutions. We prove that solutions are classical in the interior $C^\infty(\Omega)$ in x and establish a sharp $C^{s/m}(\overline{\Omega})$ regularity estimate up to the boundary.

Domingo García

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Farkas' Lemma in the non-linear setting

We prove the following Farkas' Lemma for simultaneously diagonalizable bilinear forms: if $A_1, \ldots, A_k$, and $B : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ are bilinear forms, then one —and only one— of the following holds:

(i) $B = a_1 A_1 + \cdots + a_k A_k,$ with non-negative a_i's,

(ii) there exists (x, y) for which $A_1(x, y) \geq 0, \ldots, A_k(x, y) \geq 0$ and $B(x, y) < 0$.

Moreover, we take a closer look at the reason for the lack of Farkas type results in the multilinear (and other) settings. We study evaluation maps over the space of bilinear forms and this allows us to construct new examples failing Farkas' Lemma.

The material of this talk is based on joint research with Richard Aron, Damián Pinasco and Nacho Zalduendo.

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Plays in the forest: groups acting on trees

A tree is a connected graph without cycles. Trees appear in many areas of Mathematics and Computer Science: they are examples of hyperbolic metric spaces, covering spaces of graphs, symmetric spaces of Lie groups (or rather, their p-adic analogues for primes p), to name a few.

Groups that act on trees are important not just because of the information they encode related to the context in which the trees appear, but also from the purely group-theoretic viewpoint.

I will give a (necessarily short and biased) survey of groups of automorphisms of infinite trees and try to give an idea of their relevance in contemporary research in group theory.

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A Categorical Approach to Classical and Quantum Information Geometry

Čencov and Petz's theorems are considered the cornerstone of Classical and Quantum Information Geometry. According to Čencov's theorem, the Fisher-Rao metric tensor is the only Riemannian metric tensor (up to a constant) which is invariant under congruent embeddings. An analogue problem in the quantum setting was posed by Čencov and Morozowa and fully solved by Petz, who classified all Riemannian metric tensors on the manifolds of quantum states which are monotone invariant under completely positive trace preserving maps. An interesting byproduct of this investigation is that uniqueness is lost in the quantum setting.

In this talk, a reformulation of these problems will be presented. The natural framework of this reformulation turns out to be a suitable category of non-commutative probabilities, following Čencov's original spirit. We will see how this category presents a perfect environment to discuss Čencov's and Petz's results, their unification, and their generalization to infinite dimensions.

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Coisotropic reduction in Classical Field Theories

In this talk I will review the geometric framework for Classical Field Theories, Multisymplectic Geometry. Unlike its classical analogue (symplectic manifolds), multisymplectic manifolds are heavily unexplored. In particular, there exists a vast amount of results in Sympelctic Geometry which are yet to be generalized. We will focus on presenting a recent generalization of one of these results, coisotropic reduction.

Coisotropic reduction, together with the interpretation of dynamics as a particular class of submanifolds (Lagrangian), allows for a reduction of dynamics, hence the interest in generalizing this result to Multisymplectic Geometry.

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Superreflexive spaces

In this work, the concept of superreflexivity is presented, a concept of great utility when studying geometry in Banach spaces. Various characterizations and consequences of it are provided, along with all the necessary preliminary notions to understand them. Many of these characterizations will be made from the perspective of the theory of renormings and differentiability, although others framed within the non-standard theory of Banach spaces are also presented.

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Moduli and Varieties of Group Representations

Moduli spaces of group representations arise naturally while studying certain kinds of geometric objects on manifolds. In particular, these varieties play a fundamental role in the so-called non-abelian Hodge theory.

We devote this presentation to the introduction of moduli spaces of group representations through GIT quotients, as well as some of their most celebrated invariants. In particular, we will focus on the Hodge-Deligne polynomial (which generalises the classical Hodge polynomial for non-smooth algebraic varieties) and the homotopy groups. Furthermore, we will compute these invariants for the moduli spaces of representations of the free groups F_r for any rank $r \in \mathbb{Z}_{\geq 1}$.

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Moderately discontinuous algebraic topology

In the works (1) and (2) we develop a new metric algebraic topology, called the Moderately Discontinuous Homology and Homotopy, in the context of a germ of a topological cone in $\mathbb{R}^n$ and more generally of a (degenerating) continuous family of sets in $\mathbb{R}^n \times \mathbb{R}$. This set or family is endowed with a nice metric structure such as the restriction of the euclidean metric in the ambient space, or the continuous extension of the riemannian metric in the smooth part. Then, our theory captures bilipschitz information or in other words, quasi isometric invariants, and aims to codify part of the bilipschitz geometry. For example, in the case of a topological cone (which in general is not metrically a straight cone), the moderately discontinuous theory captures the different speeds, with respect to the distance to the origin of the cone, in which the topology of the link collapses towards the origin. Similarly, in a degenerating family, it captures the different speeds of collapsing with respect to the family parameter. In this talk, I will give a gentle introduction to the theory, and explain the context in which it is originally established, but that could be enlarged in order to be applied in a more softer context.

(1) J. Fernández de Bobadilla, S. Heinze, M. Pe Pereira and E. Sampaio, *Moderately discontinuous homology.* Comm. Pure App. Math. https://doi.org/10.1002/cpa.22013. Also available in arXiv:1910.12552v3

(2) J. Fernández de Bobadilla, S. Heinze, M. Pe Pereira, *Moderately discontinuous homotopy*, IMRN. Available in ArXiv:2007.01538.

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Homomorphisms between mapping class groups

The mapping class group Map(S) of a surface S is the group of homeomorphisms of S up to homotopy. The group Map(S) is a central object in many areas of geometry such as Riemannian geometry, hyperbolic geometry, Teichmüller theory and geometric group theory. In this talk we will introduce the group Map(S), state some classical results and give a geometrical characterization of homomorphisms between mapping class groups.

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Følner-type approximations of unbounded operators motivated by Quantum Mechanics

The dichotomy amenable/paradoxical has been a central notion in group theory that has naturally evolved into other fields like pure algebras or algebras of operators on Hilbert spaces. In all these cases, amenability can be expressed as a finite (Følner type) approximation of the corresponding structure. In this talk, we will study the Følner-type approximation for bounded, and motivated by quantum mechanics, also for unbounded operators. In particular, in both settings, we will analyse the close relationship with the quasidiagonal-like approximation and show how normal, and hence, self-adjoint operators admit an intrinsic one given by its spectral resolution.These approximations will be explicitly obtained using the Weyl-Von Neumann-Berg-Sikonia Theorem stating that every normal (possibly unbounded) operator is a compact perturbation of a diagonal one.

Joint work with E. Gallardo-Gutiérrez and F. Lledó.

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Boundedness of multilinear operators at the end-points with extrapolation techniques

Extrapolation techniques allow us to determine bounds for an operator in different spaces based on known bounds on other spaces. For example, if an operator B satisfies

$$B : L^{p_1} \times L^{p_2} \longrightarrow L^p, \quad \|B\| \leq \frac{1}{(p_1-1)^\alpha (p_2-1)^\beta}$$

for some $\alpha, \beta > 0$ and every $1 < p_1, p_2 < 2$ such that $\frac{1}{p} = \frac{1}{p_1} + \frac{1}{p_2}$, we can deduce estimates in the case when $p_1 = p_2 = 1$.

The aim of this talk is to use various multilinear extrapolation techniques that extend existing linear results. This will help us understand the behaviour of operators in spaces where their boundedness is previously unknown. The techniques are based on the well-known extrapolation theorems by Yano (1) and Rubio de Francia (2), which use weights from the Muckenhoupt classes. These theorems have led to two main lines of extrapolation research that continue to this day. Recently, Carro and Domingo-Salazar (3) have combined both extrapolation techniques in the linear case to find bounds for certain operators at the "end-points". This work aims to extend their approach to the multilinear case to obtain bounds for operators like the Bochner-Riesz bilinear operator at the critical index.

(1) S. Yano, *Notes on Fourier analysis XXXIX: An extrapolation theorem.*, J. Math. Soc. Japan, **140**, 34-54, 1951.

(2) J. L. Rubio de Francia, *Factorization theory and A^p weights.*, Amer. J. Math., **106**, 533-547, 1984.

(3) M. J. Carro and C. Domingo-Salazar, *Endpoint estimates for Rubio de Francia operators.*, Trans. Amer. Math. Soc., **371**, 1621-1648, 2019.

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The topology of complex plane curve singularities

We introduce some concepts from the study of complex hypersurface singularities, such as the Milnor fiber, monodromy and Seifert form, with emphasis on the case of plane curves. We present new results obtained jointly with Pablo Portilla Cuadrado, which gives a description of these topological invariants of plane curves, and discuss some open questions.

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Zero Dynamics and Hypergeometric Representation of Sobolev-Type Discrete q-Hermite I Orthogonal Polynomials

In this work, we investigate the sequence of monic q-Hermite I-Sobolev type orthogonal polynomials of higher-order, denoted by $\{\mathbb{H}_n(x;q)\}_{n\geq 0}$, which are orthogonal with respect to the following non-standard inner product involving q-differences:

$$\langle p, q\rangle_\lambda = \int_{-1}^{1} f(x)\, g(x)\, (qx, -qx; q)_\infty d_q(x) + \lambda\, (\mathscr{D}_q^j f)(\alpha)(\mathscr{D}_q^j g)(\alpha),$$

where λ belongs to the set of positive real numbers, $\mathscr{D}_q^j$ denotes the j-th q-discrete analogue of the derivative operator, $q^j\alpha \in \mathbb{R}\backslash(-1,1)$, and $(qx, -qx; q)_\infty d_q(x)$ denotes the orthogonality weight with its points of increase in a geometric progression. Connection formulas between these polynomials and standard q-Hermite I polynomials are deduced. The basic hypergeometric representation of $\mathbb{H}_n(x;q)$ is obtained. Moreover, for certain real values of α satisfying the condition $q^j\alpha \in \mathbb{R}\backslash(-1,1)$, we present results concerning the location of the zeros of $\mathbb{H}_n(x;q)$ and perform a comprehensive analysis of their asymptotic behavior as the parameter λ tends to infinity.

Joint work with Edmundo J. Huertas, Alberto Lastra and Anier Soria-Lorente

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Reliability of Coherent Systems: Design and Stochastic Comparisons

In reliability theory, coherent systems consist of multiple interconnected components or subsystems, and play a critical role in ensuring high performance and reliability in complex systems across various industries, including telecommunications, optics, and physics. One key strategy for enhancing system reliability is redundancy, which involves duplicating critical components or subsystems to minimize the risk of failure. Redundancies can take different forms, such as active or standby, and their effective allocation is vital for designing reliable systems.

A crucial aspect of reliability analysis is the investigation of system lifetimes, especially when dependencies exist among components. The lifetime dependencies within a system significantly affect its overall reliability, and these dependencies are often modeled using copulas. Understanding and modeling these dependencies is essential for accurate reliability assessments.

Furthermore, stochastic orderings are commonly applied in reliability theory to compare the failure behavior of different systems or components. By utilizing stochastic orderings, we can determine which systems or components are more reliable or more likely to fail. In this talk, we will present key results from the literature (1-3), focusing on the design of coherent systems, the use of redundancies, and the application of stochastic orderings to improve system reliability.

This talk cover one of the topics we explore in the project TED2021-129813A-I00. The author thanks the support of MCIN/AEI/10.13039/501100011033 and the European Union "NextGenerationEU"/PRTR.

(1) N. Torrado, A. Arriaza, and J. Navarro, *A study on multi-level redundancy allocation in coherent systems formed by modules*, Reliability Engineering & System Safety, 213 (2021), 107694.

(2) N. Torrado, *Comparing the reliability of coherent systems with heterogeneous, dependent and distribution-free components*, Quality Technology & Quantitative Management, 18 (6) 2021, 740-770.

(3) N. Torrado, *Optimal component-type allocation and replacement time policies for parallel systems having multi-types dependent components*, Reliability Engineering & System Safety, 224 (2022), 108502.

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Rational functions in quantum computing

Rational functions form a very powerful building block for numerical algorithms. They outperform the usual polynomial-based algorithms for several important problems in, e.g., quantum chemistry. There is an increasing interest in developing quantum algorithms based on rational functions, since quantum computing holds the promise that one day it can compute with matrices much larger than those that can be handled by classical computers. Since no standard for computing with rational functions on a quantum computer exists, we propose two new quantum computational approaches for the construction of rational functions. Using our results, we comment on the possibility of developing quantum analogues to classical rational function-based algorithms and their corresponding challenges.

This presentation is based on joint work with Yizhi Shen, Daan Camps, Katherine Klymko and Roel Van Beeumen at Lawrence Berkeley National Laboratory, USA.