PROPUESTA DE TRABAJO DE FIN DE MÁSTER
MÁSTER EN MATEMÁTICAS AVANZADAS

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Título: A closer look at de Branges-Rovniak spaces as Dirichlet spaces.

Resumen:

Let μ be a finite positive Borel measure on $\mathbb{T} = \{z \in \mathbb{C} : |z| = 1\}$, with P_μ denoting the Poisson integral of μ. The generalized Dirichlet space $D(\mu)$ consists of all functions f in the Hardy space H^2 for which

$$\int_{\mathbb{D}} |f'(z)|^2 P_\mu(z)\, dA(z) < \infty$$

where dA denotes a normalized area measure on $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$.

For a function b in the unit ball of H^∞, the de Branges–Rovnyak space $H(b)$ is the reproducing-kernel Hilbert space consisting of analytic functions on $\mathbb{D}$ whose kernel functions have the form
$k^b_w(z) = (1 - \overline{b(w)}\, b(z))/(1 - \overline{w}\, z)$.
If $\log(1 - |b|)$ belongs to $L^1(\mathbb{T})$, there exists a unique outer function a with $a(0) > 0$ such that $|a|^2 + |b|^2 = 1$ almost everywhere on $\mathbb{T}$.

In this work, the student will study the known characterizations for the condition $H(b)=D(\mu)$ stated by C. Costara and T. J. Ransford [J. Funct. Anal. 265 (2013), no. 12, 3204–3218] and
and the recent work by Pouliasis (J. Funct. Anal. 288 (2025), no. 3, Paper No. 110717, 17 pp.)

Bibliography:

[1] O. El-Fallah, K. Kellay, J. Mashreghi, T. Ransford, A Primer on the Dirichlet Space, Cambridge Tracts in Mathematics, vol. 203, Cambridge University Press, 2014.
[2] E. Fricain, J. Mashreghi, The Theory of H(b) Spaces, vol. 1, New Mathematical Monographs, vol. 21, Cambridge University Press, 2015.
[3] E. Fricain, J. Mashreghi, The Theory of H(b) Spaces, vol. 2, New Mathematical Monographs, vol. 21, Cambridge University Press, 2015.