

PROPUESTA DE TRABAJO DE FIN DE MÁSTER
MÁSTER EN MATEMÁTICAS AVANZADAS

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Título: Higgs bundles and geometric structures

Resumen:

The theory of geometric structures on manifolds was introduced by Cartan and Ehresmann in the 1920s, following the ideas given by Klein in his Erlangen program. This theory became popular in the 1980s, when Thurston used it in the statement of his Geometrization Conjecture. Geometric structures on manifolds are also important in higher Teichmüller theory, a research area originated from the work of Hitchin and others.

It is well-known that the Teichmüller space of a compact surface can be identified with a connected component of the character variety of representations of the fundamental group of the surface in $\mathrm{PSL}(2, \mathbb{R})$. Higher Teichmüller spaces are generalizations of this, which exist in the moduli space of representations of the fundamental group of the surface into certain real simple non-compact Lie groups of higher rank. As for the usual Teichmüller space, these spaces consist entirely of discrete and faithful representations. In [1], the authors give a classification of all possible higher Teichmüller spaces, and a parametrization of them from a Higgs bundle perspective. For a survey of Higgs bundles in this context one may look at [2].

Higgs bundles are an important tool in higher Teichmüller theory since they can be used to describe the topology of the character variety. Although they were initially believed to give very little information on the geometry of a single representation, there are now many examples where the Higgs bundle can be used to give interesting information on the geometric structures associated with a certain representation of a surface group. A good account of some of these developments is given in [3].

The main objectives of this TFM are:

(1) Review the theory of Higgs bundles in connection to higher Teichmüller spaces along the lines of [1] and [2].

(2) Following [3], explore how the theory of Higgs can be used to study geometric structures coming from surface group representations, especially from representations in higher Teichmüller spaces for groups of small rank.

References

- [1] S. Bradlow, B. Collier, O. García-Prada, P. Gothen and A. Oliveira, A general Cayley correspondence and higher rank Teichmüller spaces, *Annals of Mathematics*, 200(3) (2024), 803–892.
- [2] O. García-Prada, Higgs bundles and higher Teichmüller spaces, *Handbook on Teichmüller theory*, Athanase Papadopoulos (editor), European Mathematical Society, 2020.
- [3] D. Alessandini, Higgs Bundles and Geometric Structures on Manifolds, *SIGMA* 15 (2019).