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Título: Moment map equations in Kähler geometry and gauge theory

Resumen:

Moment map equations in Kähler geometry and gauge theory

A fundamental set of equations in the context of gauge theory on complex manifolds is given by the Hermitian Yang-Mills equations on a holomorphic vector bundle over a compact Kähler manifold. Their study gives rise to the Hitchin-Kobayashi correspondence proved by Donaldson and Uhlenbeck-Yau. Another paradigmatic set of equations is given by the Kähler-Einstein equations and more generally the constant scalar curvature condition for a Kähler metric on a compact complex manifold. All the above mentioned equations have a moment map interpretation in the sense of symplectic geometry. Taking this symplectic point of view, a programme to study coupled equations for Yang-Mills connections and Kähler metrics was initiated more than fifteen years ago by Álvarez-Cónsul, García-Fernández and García-Prada [1] with the introduction of the Kähler-Yang-Mills equations.

The fruitful interplay between Hamiltonian systems with symmetry and complex geometry is of paramount importance in symplectic geometry in finite dimensions. A particularly powerful tool in connecting these areas is the Kempf-Ness theorem [2] which describes the equivalence between the notions of quotient in symplectic and algebraic geometry. The theorem states that the symplectic quotient of a Hamiltonian action by a compact Lie group is isomorphic to the GIT quotient of the associated action of the complexified group.

In [3], Diez, Futaki and Ratiu propose an approach to generalize Kempf-Ness theory to infinite dimensions, invoking the fundamental role played by the Maurer-Cartan form. This approach allows them to define and study objects essential for the Kempf-Ness theorem, such as the complex model for orbits and the Kempf-Ness function, as well as establishing its convexity properties and defining a generalized Futaki character. They show how their framework can be applied to the study of various problems in Kähler geometry, and gauge theory.

The main objectives of the proposed TFM are the following:

(1) Study the moment map equations appearing in [1]

(2) Study the basics of the approach in [3] to the infinite dimensional Kempf-Ness

(3) Explore whether the theory developed in [1] fits into the scheme of [3]

(4) An alternative objective could be to explore the same for the theory described in [4] and [5]

References:

[1] L. Álvarez-Cónsul, M. Garcia-Fernandez and O. García-Prada, Coupled equations for Kähler metrics and Yang–Mills connections, Geom. Topol. 17 (2013) 2731–2812.

[2] G. Kempf and L. Ness. The length of vectors in representation spaces. In: Algebraic Geometry (Proc. Summer Meeting, Univ. Copenhagen, Copenhagen, 1978). Vol. 732. Lecture Notes in Math. Springer, Berlin, 1979, pp. 233–243.

[3] T. Diez, A. Futaki and T. S. Ratiu, Cartan Geometry and Infinite-Dimensional Kempf–Ness Theory, arXiv

[4] O. García-Prada and D. Salamon. A moment map interpretation of the Ricci form, Kähler–Einstein structures,
and Teichmüller spaces. In: Integrability, Quantization, and Geometry Ii. Quantum Theories and Algebraic Geometry. Dedicated to the Memory of Boris Dubrovin 1950–2019. Providence, RI: American Mathematical Society (AMS), 2021, pp. 223–255.

[5] O. García-Prada, D. Salamon, and S. Trautwein. Complex structures, moment maps, and the Ricci form. Asian J.
Math. 24 (2020), pp. 821–854.