PROPUESTA DE TRABAJO DE FIN DE MÁSTER
MÁSTER EN MATEMÁTICAS AVANZADAS

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Título: Subnormal operators

Resumen:

The study of **subnormal operators** lies at the intersection of operator theory, complex analysis, and functional analysis. These operators generalize normal operators and play a central role in understanding spectral theory, invariant subspaces, and function-theoretic operator models. Recall that an operator $T \in \mathcal{B}(\mathcal{H})$is called subnormal if it admits a normal extension, that is, if there exists a Hilbert space $\mathcal{K} \supseteq \mathcal{H}$and a normal operator $N \in \mathcal{B}(\mathcal{K})$such that $T = N|_{\mathcal{H}}$.

After reviewing the necessary background on Hilbert space operators and the spectral theorem for normal operators, the master thesis will develop the main characterizations of subnormality. In particular, the Bram–Halmos positivity condition and its connection to moment problems, as well as the functional model of cyclic subnormal operators as multiplication by the coordinate function on $P^2(\mu)$, the closure of polynomials in $L^2(\mu)$for a suitable positive Borel measure μ(the Berger measure in the case of weighted shifts).

The central result will discuss Scott Brown theorem, which states that every non-normal subnormal operator admits a non-trivial invariant subspace. This theorem provides a significant positive contribution to the invariant subspace problem within the class of subnormal operators. The proof relies on the functional model, analytic approximation techniques, and structural properties of the associated measure.

Finally, the goal would be to place Scott Brown theorem within the broader context of dilation theory, hyponormal operators, and ongoing research surrounding invariant subspaces.

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