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Título: Quasitriangular Operators

Resumen y bibliografía:

This work aims to study the class of quasitriangular operators acting on a separable Hilbert space. Throughout the master thesis, several characterizations and fundamental properties of this class of operators will be presented, highlighting how quasitriangular operators arise naturally as a generalization of triangularizable operators in (M_n(\mathbb{C})).

Furthermore, the relevance of quasitriangular operators to the Invariant Subspace Problem will be analyzed. In particular, based on a series of seminal papers by Apostol, Foiaș, and Voiculescu, it is known that every operator which is not quasitriangular admits a non-trivial invariant subspace. This result underscores the central role played by quasitriangular operators in the study of this longstanding problem.

A substantial part of the analysis will be guided by the use of the Calkin algebra as a key tool for proving several theorems that facilitate the investigation of specific subclasses of quasitriangular operators, such as normal operators perturbed by compact operators. In this context, classical results including the Brown–Douglas–Fillmore theorem and the Weyl–von Neumann–Berg–Sikonia theorem will be discussed.

The overall objective of this master thesis is to provide a precise and rigorous framework for understanding how to construct and identify the relevant projections associated with specific quasitriangular operators, thereby contributing to a deeper structural and practical understanding of this class of operators.

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