Modeling, Simulation and Optimization of a Polluted Water Pumping Process in Open Sea.

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Resumen

The objective of this article is to find the optimal trajectory of a pumping ship, used to clean oil spots in the open sea, in order to pump the maximum quantity of pollutant on a fixed time period. We use a model previously developed to simulate the evolution of the oil spots concentration due to the coupling of diffusion, transport from the wind, sea currents and pumping process and reaction due to the extraction of oil. The trajectory of the ship is directly modeled by considering a finite number of interpolation points for cubic splines. The optimization problem is solved by using a global optimization algorithm based on a hybrid Genetic Algorithm. Finally, we check the efficiency of our approach by solving a numerical example considering based on real coefficients.

Keywords: Sea pollution; Pumping ship; Reaction-Advection-Diffusion model; Optimal trajectory; Global Optimization.

1. Introduction

Recent oil contamination hazards in the open sea (see [20, 24]), shows the importance of finding solutions to remove the oil in an efficient way. To do that, there exist a large number of cleaning technologies [22]. Here, we focus in the use of a pumping ship to clean the oil contaminated water [2, 21]. More precisely, given a particular oil contamination scenario during a fixed time interval, we are interested in finding an optimal trajectory for this pumping ship in order to find an optimum cleaning process.

In order to solve this complex optimization problem, using mathematical and computational methods, we need to model first the evolution of the oil spots concentration resulting from the combined effects of diffusion, transport (by wind and sea currents) and the action of the pumping ship (that implies transport and a reaction phenomena). In this paper we use a finite volume numerical model previously developed [1].

It is also necessary to formulate mathematically our optimization problem. In particular, we need to model the ship trajectory by the use of a continuous function generated by cubic spline interpolation, where the position of the finite number of interpolation points are the optimization variables. The objective function is designed to maximize the amount of oil pumped during the fixed time interval.

Since this optimization problem seems to have various local and global minima [8], we solve it by considering an hybrid Genetic Algorithm [3, 8, 12].

To verify the efficiency of our approach, we consider and solve numerically a particular example based on real coefficient values [20].

In Section 2, we introduce the numerical model considered to simulate the movement of the oil spots and the effect of the pumping ship. Section 3 presents the optimal trajectory problem. Finally, in Section 4, we show the numerical results over the considered example.

2. Mathematical model for oil spots movement in the open sea

Here, we present a numerical model used to simulate the evolution of the oil spots concentration, due to the effects of the sea, wind and pumping process and previously introduced in [1]. First we introduce the continuous equations. Then, this model is discretized by considering a Finite Volume approach.

2.1. Continuous model

We consider a spatial domain $\Omega = (x_{\min}, x_{\max}) \times (y_{\min}, y_{\max}) \subset \mathbb{R}^2$, large enough to ensure that the pollutant will stay in Ω during the corresponding fixed time interval $(0, T)$.

We assume that the density of the pollutant is smaller than the one of the sea water (so that it remains at the top) and the layer-thickness of the pollutant is a known constant h [19].

We denote by $c(x, t)$ the pollutant superficial concentration, measured as the volume of pollutant per surface area at $\{x, t\} \in \Omega \times (0, T)$. We assume that the evolution of c is governed by five main effects, namely:

- Diffusion of the pollutant
- Transport due to the wind
- Transport due to the sea currents
- Transport and sink due to the pumping process

Furthermore, we consider that the pumping ship follows a trajectory $\gamma(t) \in C^0([0,T],\Omega), t \in [0,T]$, that remains inside the region Ω and the pump is a cylinder with a cross section of radius R_p and height h_p (we suppose $h_p \geq h$), that pumps the fluid at a velocity Q in the radial directions.

Under these assumptions, the space-time distribution of c is governed by the following reaction-advection-diffusion type system [1, 11] (the existence, uniqueness and continuous dependence with respect to data of the solution of this system is mathematically well known [16]):

$$\left\{ \begin{array}{ll} \dfrac{\partial c}{\partial t} - \nabla \cdot \mathbf{d} \nabla c + \nabla \cdot c\,\mathbf{w} + \nabla \cdot c\,\mathbf{s} & \\ \qquad\qquad + \nabla \cdot c\,\mathbf{p} = -\dfrac{2Q}{R_p}\, c\ \chi_{B(\gamma(t),R_p)}, & \text{in } \Omega \times (0,T), \\ c = 0, & \text{on } \partial\Omega \times (0,T), \\ c = c_0, & \text{in } \Omega \times \{0\}, \end{array} \right. \tag{1}$$

where:

- $B(\gamma(t), R_p)$ is the ball of center $\gamma(t)$ and radius R_p,
- $\mathbf{p}(\xi,t) = \left\{ \begin{array}{ll} QR_p \dfrac{\overrightarrow{\gamma(t)\xi}}{|\overrightarrow{\gamma(t)\xi}|^2}, & \text{if } \xi \in \Omega \backslash B(\gamma(t),R_p), \\ 0, & \text{if } \xi \in B(\gamma(t),R_p), \end{array} \right.$ see details in [1].
- $\chi_{B(\gamma(t),R_p)}(\xi) = \left\{ \begin{array}{ll} 0, & \text{if } \xi \in \Omega \backslash B(\gamma(t),R_p), \\ 1, & \text{if } \xi \in B(\gamma(t),R_p), \end{array} \right.$
- function $c_0 : \Omega \to \mathbb{R}$ is the initial superficial concentration, which we assume with compact support in Ω,
- $\mathbf{d} = \begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}$ and $d_1, d_2 > 0$ are the diffusion coefficients in the west-east and south-north directions,
- $\mathbf{w}$ is the horizontal components of the wind velocity multiplied by a suitable drag factor,
- $\mathbf{s}$ is the sea current velocity.

2.2. Numerical approximation model

A Finite Volume numerical method (see [5, 6, 17] for implementation and convergence results) has been used to approximate numerically the solution of the continuous model presented in 2.1 [1]. More precisely, given $I, J \in \mathbb{N}$ we divide $\Omega = (x_{\min}, x_{\max}) \times (y_{\min}, y_{\max})$ into control volumes $\Omega_{i,j}$. For $i = 1, \ldots, I$; $j = 1, \ldots, J$, we define

$$\Omega_{i,j} = (x_{\text{mín}} + (i-1)\Delta x, x_{\text{mín}} + i\Delta x) \times (y_{\text{mín}} + (j-1)\Delta y, y_{\text{mín}} + j\Delta y), \tag{2}$$

with $\Delta x = \dfrac{x_{\text{máx}} - x_{\text{mín}}}{I}$ and $\Delta y = \dfrac{y_{\text{máx}} - y_{\text{mín}}}{J}$. We define $\Delta t = \dfrac{T}{N}$, where $N \in \mathbb{N}$ is the number of time steps.

Considering a fully implicit time discretization of backward Euler type for the time discretization of (1) with an upwind scheme for the transport term, one obtains at $t = n\Delta t$ on the cell $\Omega_{i,j}$, for $i = 1, \ldots, I$ and $j = 1, \ldots, J$, the following scheme:

$$C^0_{i,j} = C_0(\xi_{i,j}), \;\; \xi_{i,j} \text{ being the center of cell } \Omega_{i,j}; \tag{3}$$

for $n \geq 0$ we compute $\{C^n_{i,j}\}$ (with $C^n_{i,j} \approx c(n\Delta t, \xi_{i,j})$) from $\{C^{n-1}_{i,j}\}$ using :

$$\begin{aligned}
&\frac{C^n_{i,j} - C^{n-1}_{i,j}}{\Delta t} + 2\left(\frac{d_1}{(\Delta x)^2} + \frac{d_2}{(\Delta y)^2}\right) C^n_{i,j} \\
&-\frac{d_1}{(\Delta x)^2}\left(C^n_{i+1,j} + C^n_{i-1,j}\right) - \frac{d_2}{(\Delta y)^2}\left(C^n_{i,j+1} + C^n_{i,j-1}\right) \\
&+\frac{1}{\Delta x}[\text{máx}(0, V^n_{x,i,j-\frac{1}{2}})C^n_{i,j} + \text{mín}(0, V^n_{x,i,j-\frac{1}{2}})C^n_{i+1,j} \\
&\qquad - \text{máx}(0, V^n_{x,i-1,j-\frac{1}{2}})C^n_{i-1,j} - \text{mín}(0, V^n_{x,i-1,j-\frac{1}{2}})C^n_{i,j}] \\
&+\frac{1}{\Delta y}[\text{máx}(0, V^n_{y,i-\frac{1}{2},j})C^n_{i,j} + \text{mín}(0, V^n_{y,i-\frac{1}{2},j})C^n_{i,j+1} \\
&\qquad - \text{máx}(0, V^n_{y,i-\frac{1}{2},j-1})C^n_{i,j-1} - \text{mín}(0, V^n_{y,i-\frac{1}{2},j-1})C^n_{i,j}] \\
&+\frac{2\pi R_p Q}{\Delta x \Delta y} C^n_{i,j}\chi^{p,n}_{i,j} = 0,
\end{aligned} \tag{4}$$

where in (4)

- $C^n_{k,l} = 0$ if $k \in \{0, I+1\}$ or $l \in \{0, J+1\}$,
- $\Omega_{i_{p,n},j_{p,n}}$ is the cell containing $\gamma(n\Delta t)$, $\chi^{p,n}_{i,j} = 0$ if $\{i,j\} \neq \{i_{p,n}, j_{p,n}\}$ and $\chi^{p,n}_{i,j} = 1$ if $\{i,j\} = \{i_{p,n}, j_{p,n}\}$ (if $\gamma(n\Delta t)$ is in the boundary of several cells we choose the cell of larger index),
- $\mathbf{V}(\xi,t) = (V_x(\xi,t), V_y(\xi,t)) = \mathbf{w}(\xi,t) + \mathbf{s}(\xi,t) + \mathbf{p}(\xi,t)$, with $\xi \in \Omega$ and $t \in [0,T]$,
- $V^n_{x,i,j-\frac{1}{2}} = V_x((x_{\text{mín}} + i\Delta x, y_{\text{mín}} + (j - \frac{1}{2})\Delta y), n\Delta t)$,
- $V^n_{y,i-\frac{1}{2},j} = V_y((x_{\text{mín}} + (i - \frac{1}{2})\Delta x, y_{\text{mín}} + j\Delta y), n\Delta t)$.

The solution of the non symmetric linear system (4) is obtained by using a stabilized Bi-Conjugate gradient type algorithm [1, 15, 23].

3. Optimal trajectory

As mentioned in Section 1, we address the problem of finding an optimal trajectory for the pumping ship, for a particular oil contamination scenario during a fixed time interval $(0, T)$.

For the given time T, we minimize the concentration $c(\xi, T)$ of the remaining pollutant in Ω, which is equivalent to maximize the amount of pumped oil from the sea. More precisely, we are interested in solving the following optimization problem:

$$\min_{\gamma \in D_c} J_c(\gamma) \tag{5}$$

where $J_c(\gamma) = \iint_\Omega c(0, x)\mathrm{d}x - \int_0^T c(\tau, \gamma(\tau))Q\mathrm{d}\tau$ is the objective function, $D_c = \{\gamma \in C^1([0, T], \Omega)$ such that $|\gamma'(t)| \leq V_{\text{máx}}, \forall t \in [0, T]\}$ is the feasible region and $V_{\text{máx}}$ is the maximum velocity of the ship when performing the pumping process. This restriction on the length of γ avoids to consider trajectories implying non realistic ship velocities.

In order to find numerically a smooth optimal pump trajectory (i.e. without sharp corners), we consider trajectories built by using cubic spline interpolation through $n_{\text{npi}} \in \mathbb{N}$ 2-D interpolation points.

The set of interpolation points, denoted by P_{int}, is constructed by using a polar representation:

$$P_{\text{int}} = \{(r_1, \theta_1), ..., (r_{n_{\text{npi}}}, \theta_{n_{\text{npi}}})\},$$

where $r_i \in [0, r_{\text{máx}}]$, with $r_{\text{máx}} = V_{\text{máx}} * (T/n_{\text{npi}})$ (modeling the ship velocity constraint), and $\theta_i \in [0, 2\pi)$, for $i = 1, ..., n_{\text{npi}}$.

Given an interpolation point expressed in Cartesian coordinates (x_k^{int}, y_k^{int}), with $k \in \{1, ..., n_{\text{npi}} - 1\}$, the next one $(x_{k+1}^{\text{int}}, y_{k+1}^{\text{int}})$ is built as:

$$x_{k+1}^{\text{int}} = x_k^{\text{int}} + r_k \cos(\theta_k),$$

$$y_{k+1}^{\text{int}} = y_k^{\text{int}} + r_k \sin(\theta_k).$$

The resulting interpolated trajectory is denoted by γ or (γ_x, γ_y) or $\gamma_{(r_i, \theta_i)}$.

Furthermore, we need to avoid the ship leaving the domain of study Ω. To accomplish this, we project the trajectory γ using an orthogonal projector on Ω, called Pr_Ω, defined as:

$$\begin{aligned}\text{Pr}_\Omega(\gamma_{(r_i,\theta_i)}(\tau)) = \Big(&\text{máx}(\text{mín}(\gamma_x(\tau), x_{\max}), x_{\min}), \\ &\text{máx}(\text{mín}(\gamma_y(\tau), y_{\max}), y_{\min})\Big).\end{aligned} \tag{6}$$

Thus, the numerical optimization problem that we solve, is of the form:

$$\begin{array}{l} \qquad\qquad \text{mín}\, J(r_i, \theta_i) \\ \text{subject to} \\ \quad 0 \leq r_i \leq r_{\text{máx}} \quad, i = 1, ..., n_{\text{npi}}, \\ \quad 0 \leq \theta_i < 2\pi \quad, i = 1, ..., n_{\text{npi}}, \end{array} \tag{7}$$

where $J(r_i, \theta_i) = \iint_\Omega c(0, x)\mathrm{d}x - \int_0^{T_o} c(\tau, \gamma_{(r_i,\theta_i)}(\tau))Q\mathrm{d}\tau$ is the objective function and $\{(r_i, \theta_i)\}_{i=1}^{n_{\mathrm{npi}}} \subset D$ are the discrete optimization variables with $D = [0, r_{\text{máx}}] \times [0, 2\pi)$ is the feasible region. The total number of optimization variables is $N = 2n_{\mathrm{npi}}$.

Since problem (7) has many local and global minima [8], we need to use a global optimization method capable to find the global solution. Here, we use the *Global Optimization Platform* software (*GOP*), freely available at `http://www.mat.ucm.es/momat/software.html`, with a genetic algorithm [9, 10] as the core algorithm and where the initial population is optimized by using a multi-layer secant method [12]. The algorithm parameters used during this work are given in [8]. A complete description and validation of this algorithm can be found in [4, 13, 14].

4. Numerical experiments

In this section, we check the efficiency of our approach by considering a particular numerical example.

4.1. Numerical example

We have created a representative example by considering reasonable (although fictitious) values for the model parameters based on literature [2, 18, 19]. The considered parameters are the following:

- The computational domain Ω is defined by $x_{\min} = 0$ m, $x_{\max} = 2 \times 10^4$ m, $y_{\min} = 0$ m and $y_{\max} = 2 \times 10^4$ m.
- The constraint $r_{\text{máx}} = 1000$ m.
- The number of interpolation points is $n_{\mathrm{npi}} = 10$.
- The simulation time is equal to one day: $T = 86400$ s.
- We consider a discretization mesh of (I, J)=$(50, 50)$.
- The time step is $\Delta t = 172{,}8$ s (i.e. N =500).
- The diffusion coefficients are $d_1 = d_2 = 0{,}5$ m^2/s.
- The pump parameters are $Q = 100$ m/s and $R_p = 1$ m.

We consider two circular spots defined by:

$$c(\xi, 0) = \chi_{B((8000,8000),1200)}(\xi) + \chi_{B((8000,12000),1200)}(\xi). \tag{8}$$

The wind multiplied by a drag factor plus the sea velocity field, $\mathbf{s}(\xi, t) + \mathbf{w}(\xi, t)$, is defined by

$$\Big(\frac{x}{4x_{\max}} \cos(\frac{\pi t}{3600}), \frac{y}{4y_{\max}} \sin(\frac{\pi t}{3600})\Big), \tag{9}$$

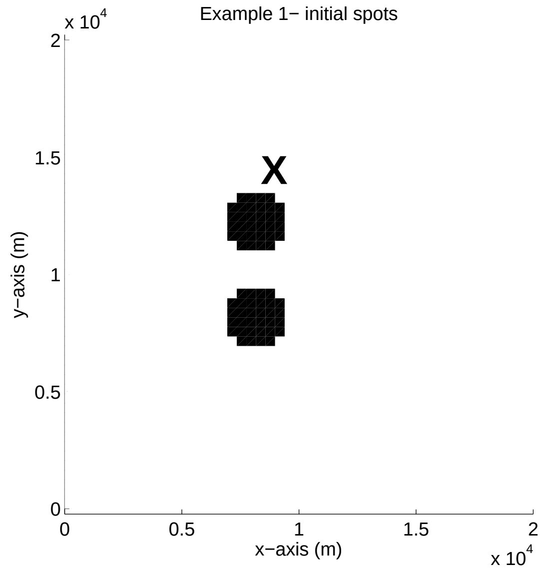

Figura 1: Initial position of the pollutant spots (in black the concentration value is 1 whereas in white is 0) in the domain Ω for the considered example introduced in Section 4.1. The initial position (**X**) of the pump is also shown.

for $t \in [0, T]$ and $\xi = (x, y) \in \Omega$.

The initial position of the pump is set to (10000, 14000). The initial pollutant concentration and initial position of the pump are depicted in Figure 1.

Furthermore, we have designed a fixed trajectory crossing the initial oil spots at constant velocity, as a reference to compare with the optimal trajectory obtained by using the hybrid Genetic Algorithm. This fixed trajectory is depicted by Figure 2.

4.2. Results

We have used a quad-core computer 64-Bit PC of 2.8Ghz and 12 GB of local memory. The code is programmed in Fortran 90. Double precision values were used in all computations. Each cost function evaluation takes around 1 second. The total optimization process requires 48000 seconds.

The resulting optimal and fixed trajectories, and their respective final oil concentration distributions, are depicted in Figure 2 (which can be compared with the initial concentrations in Figure 1). We point out that the gray-scale has been modified in order to emphasize the difference between the concentration distribution for the optimal and for the fixed trajectories.

The final percentage of remaining oil obtained with the optimal and fixed trajectories is 1.98 and 12.34, respectively. We can observe that the percentage of the remaining oil for the optimal trajectory is sufficiently lower than the fixed one. This shows the efficiency of our approach.

5. Conclusions

In this work, we have used a novel model, as reported in [1], to simulate the evolution of oil spots in the open sea considering the wind and sea currents and the effect of a pumping ship used to clean it.

We have modeled the trajectories considering cubic spline interpolation techniques. Those interpolation points are used as the independent variables for an optimization problem designed to maximize the amount of pumped oil during a fixed time interval. We have used an hybrid Genetic Algorithm to solve our optimal trajectory problem.

This approach has been validated by considering a numerical example. The obtained results for simulations show the efficiency of our approach. The developed tool can then be used for real cases.

A full description of this work can be found in [1, 8].

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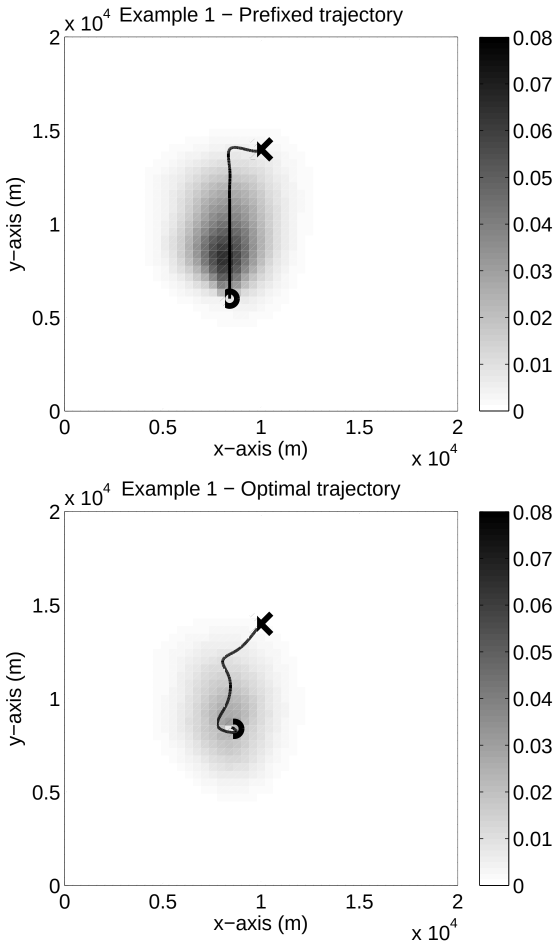

Figura 2: Final concentration obtained by considering the prefixed (**Left**) and optimal (**Right**) trajectories for the considered example introduced in Section 4.1.. The initial position (**X**), the final position (**o**) and trajectory (–) of the pump are also shown.

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