

Historia de los Poliedros

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Seminario de Historia de las Matemáticas 2016–17





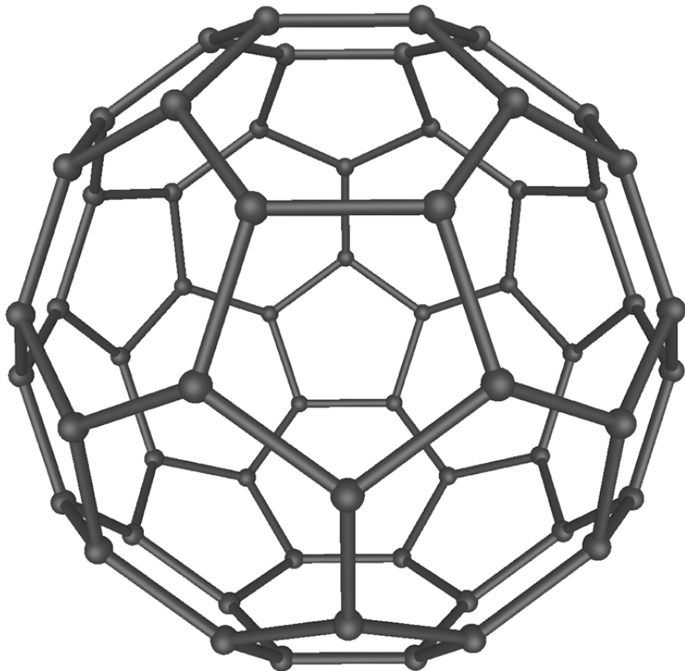




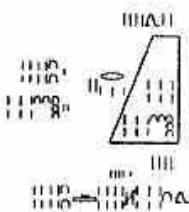




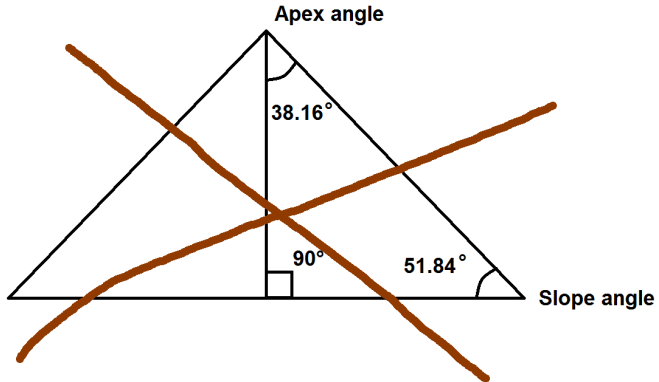








$$V = \frac{h}{3}(a^2 + ab + b^2)$$



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Triángulo de la derecha ¿isósceles? **¡Ojo con las falsedades de Internet!**

Griegos y poliedros

Demócrito (de Abdera), finales s. V a.C.

Eudoxo (de Cnido), s. IV a.C.

Teeteto (de Atenas), s. IV a.C.

Platón (de Atenas), s. V y IV a.C.

Euclides (de Alejandría), s. IV y III a.C.

Arquímedes (de Siracusa), s. III a.C.

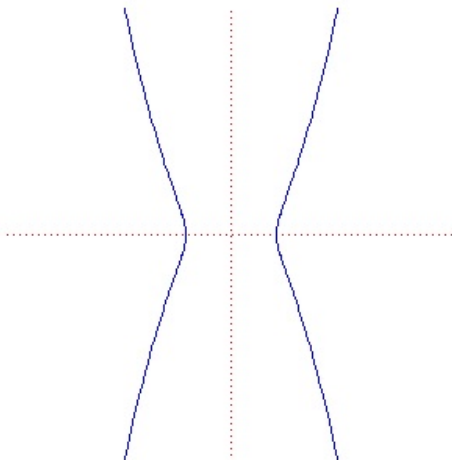


Pappus (de Alejandría), s. III d.C.

Demócrito (fin s.V a.C.) y Eudoxo (409–356 a.C. aprox)

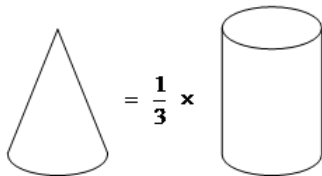
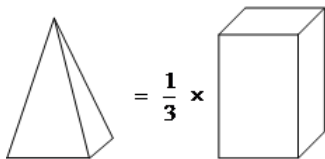


Kampyle of Eudoxus



$$a^2x^4 = b^4(x^2 + y^2) \quad \text{Campila de Eudoxo}$$

Demócrito (fin s.V a.C.) y Eudoxo (409–356 a.C. aprox)



$$V = \frac{Bh}{3}$$

Teeteto (415–369 a.C. aprox) y Platón (427–347 a.C.)

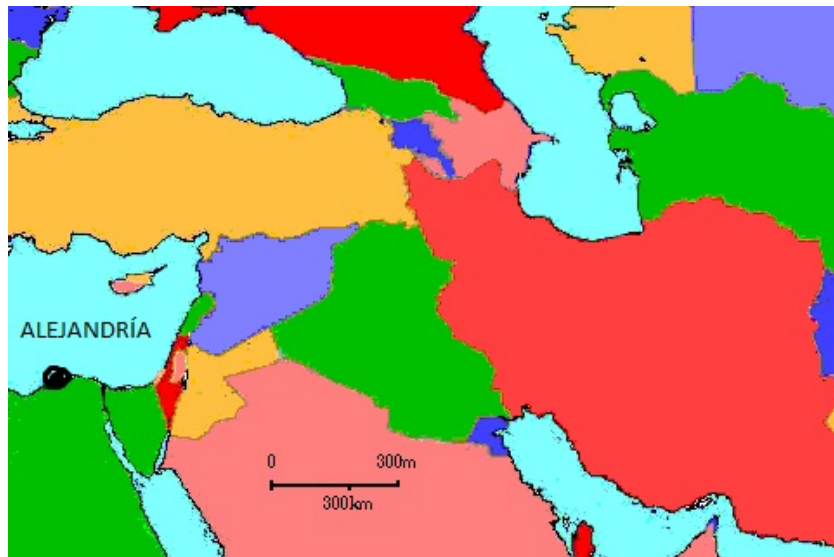


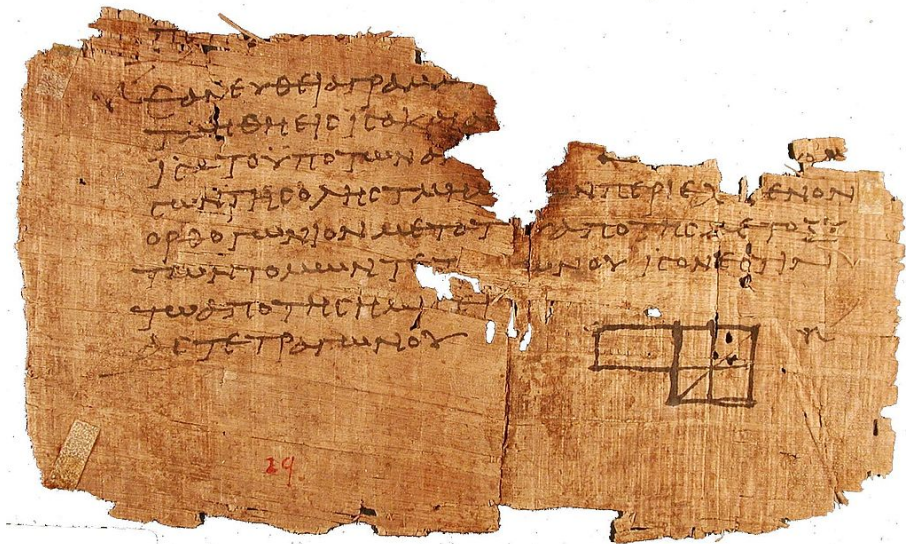






Euclides (323–285 a.C. aprox. activo)



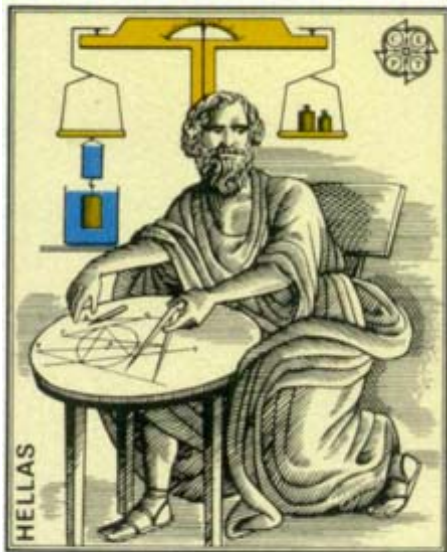




Arquímedes (287–212 a.C. aprox)

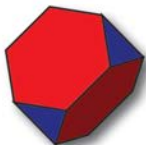


EUROPA CEPT 1983



ΕΛΛΗΝΙΚΗ ΔΗΜΟΚΡΑΤΙΑ 80

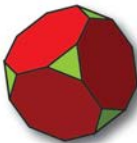




TRUNCATED TETRAHEDRON



CUBOCTAHEDRON



TRUNCATED CUBE



TRUNCATED OCTAHEDRON



RHOMBICUBOCTAHEDRON



TRUNCATED CUBOCTAHEDRON



SNUB CUBE



ICOSIDODECAHEDRON



TRUNCATED DODECAHEDRON



TRUNCATED ICOSAEDRON



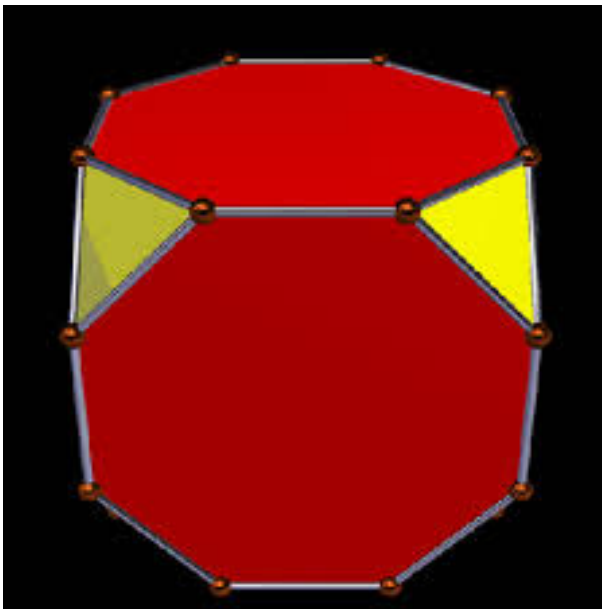
RHOMBICOSIDODECAHEDRON

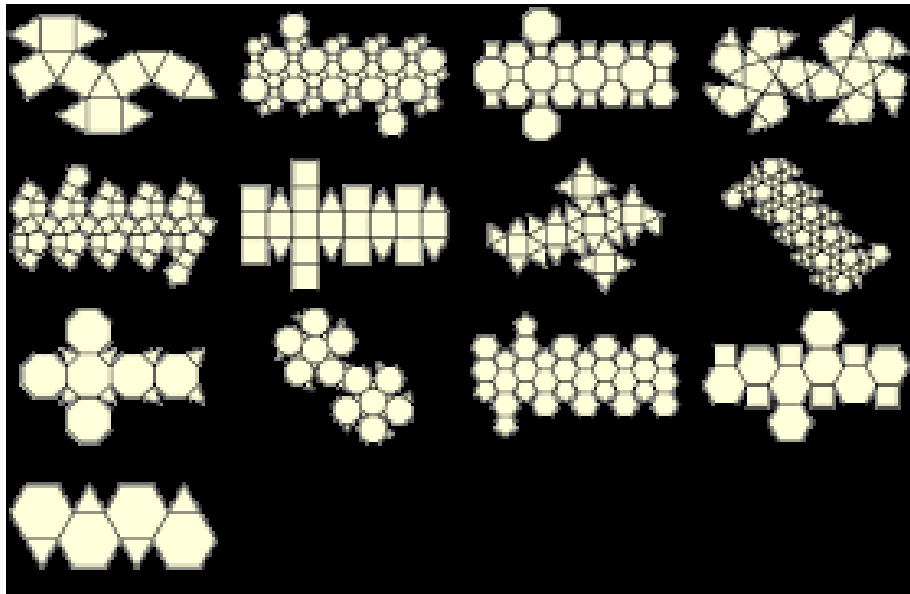


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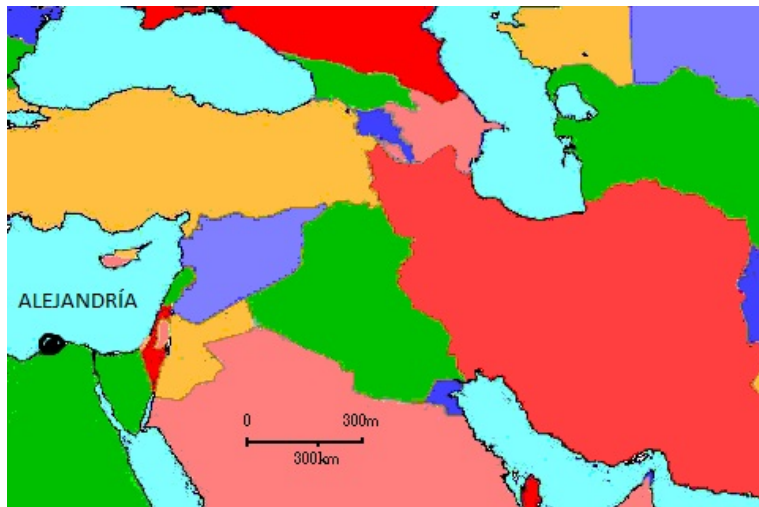
SNUB DODECAHEDRON

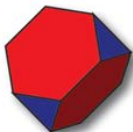






Pappus (290–350 d.C. aprox)





TRUNCATED TETRAHEDRON



CUBOCTAHEDRON



TRUNCATED CUBE



TRUNCATED OCTAHEDRON



RHOMBICUBOCTAHEDRON



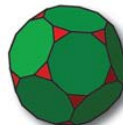
TRUNCATED CUBOCTAHEDRON



SNUB CUBE



ICOSIDODECAHEDRON



TRUNCATED DODECAHEDRON



TRUNCATED ICOSAHEDRON



RHOMBICOSIDODECAHEDRON



TRUNCATED ICOSIDODECAHEDRON



SNUB DODECAHEDRON



Durero (1471–1528)





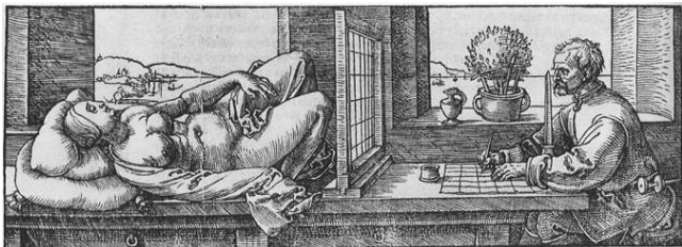


Fig. 34. Albrecht Dürer, "A Draftsman Making a Perspective Drawing of a Woman." Woodcut. From *Underweysung der Messung*. All rights reserved, The Metropolitan Museum of Art. Gift of Felix M. Warburg, 1918. (18.58.3 [recto]).

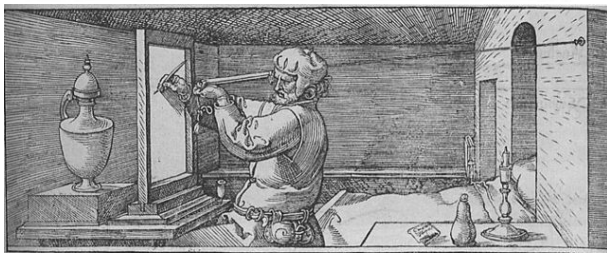


Fig. 32. Albrecht Dürer, "A Man Drawing an Urn." Woodcut. From *Underweysung der Messung*. All rights reserved, The Metropolitan Museum of Art, Harris Brisbane Dick Fund, 1941. (41.48.3).

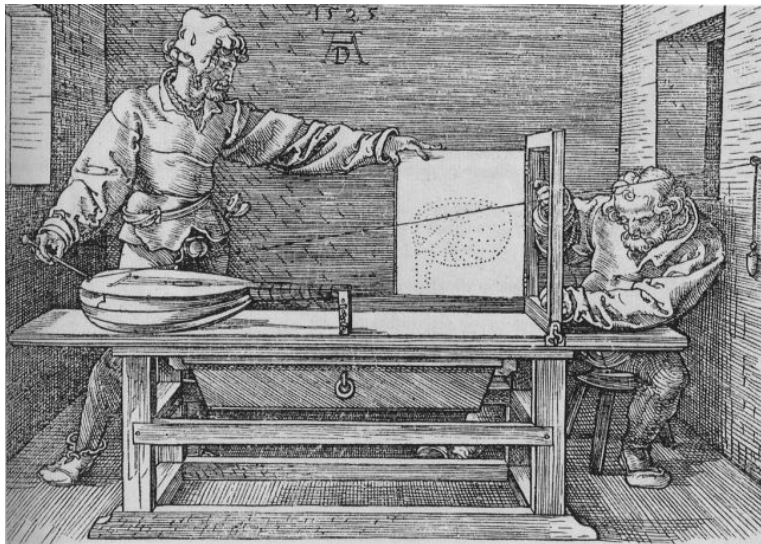
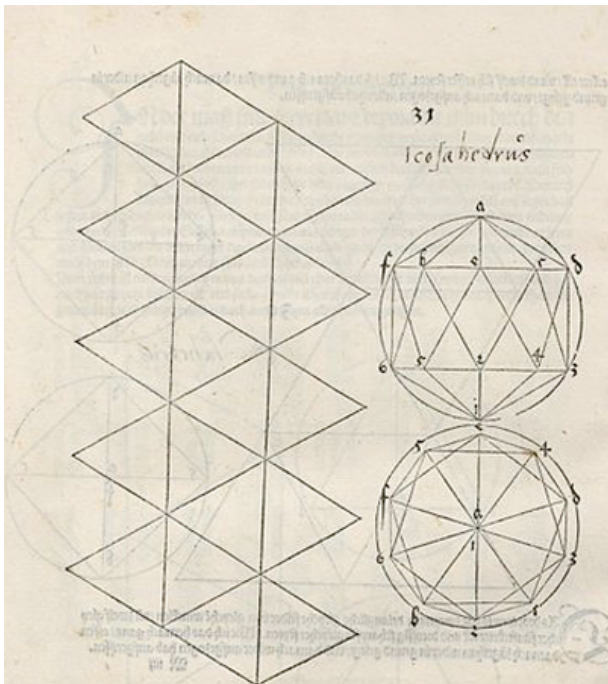


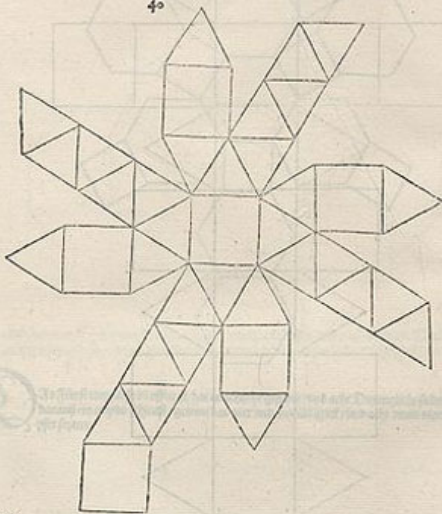
Fig. 31. Albrecht Dürer, "Draughtsman Drawing a Lute." Woodcut. Collection Centre Canadien d'Architecture/Canadian Centre for Architecture, Montréal.

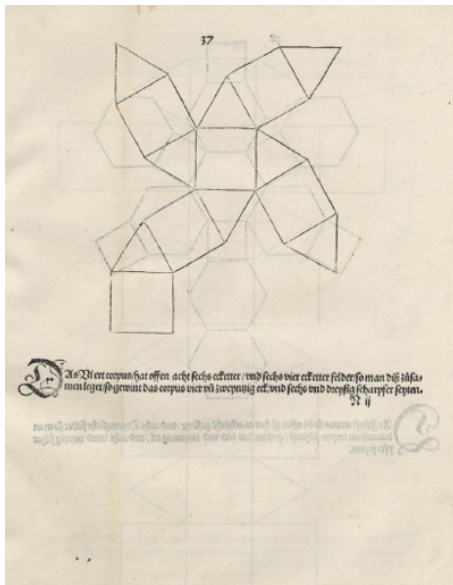




Das Sechß eckig so das außsichan weirt hat es sechs gesichte/ und inder dreyßig Denangh
 die fider/ so man das zusamen leget. gewint es vier ründ inder dreyßig al/ ründ sechßig schen
 pfer seyen.

40



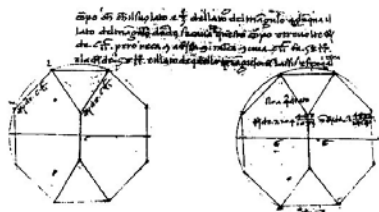
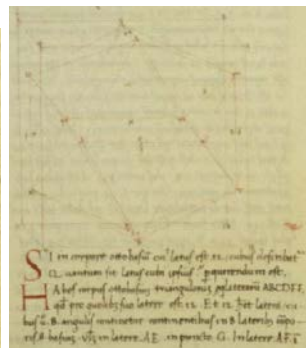
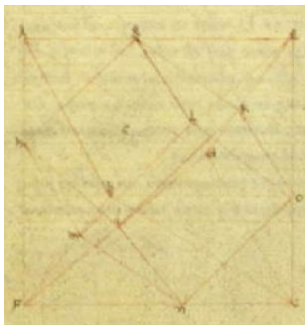


¿Existe siempre un desarrollo? ¡Problema abierto! (Problema de Durero o Conjet. de Shephard)

Artistas del Renacimiento (Italia) Paolo Uccello, Piero della Francesca, Luca Pacioli, L.da Vinci etc.

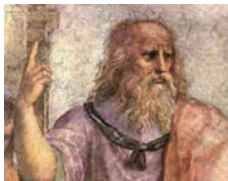
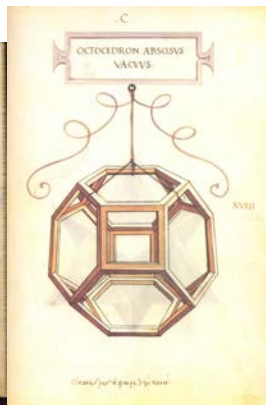
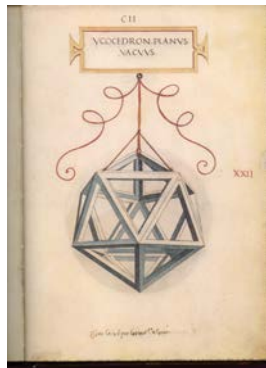


Uccello



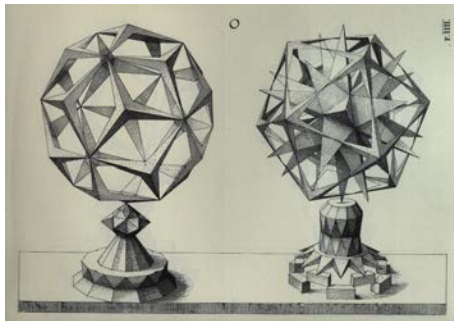
y fórmula volumen tetraedro arbitrario

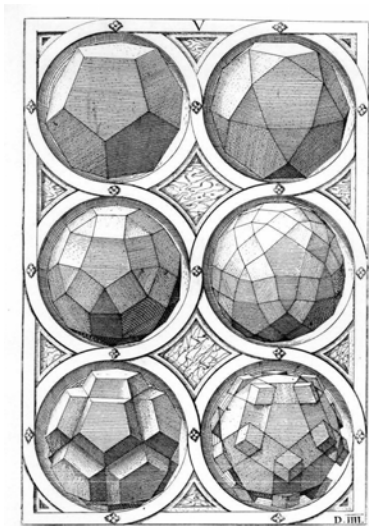
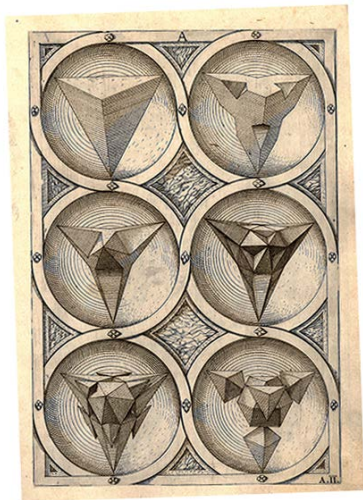
Leonardo da Vinci



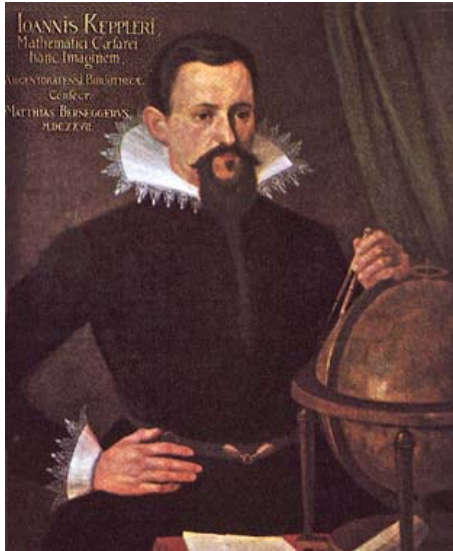
Maestros alemanes: Stöer, Jamintzer etc., s. XVI y XVII

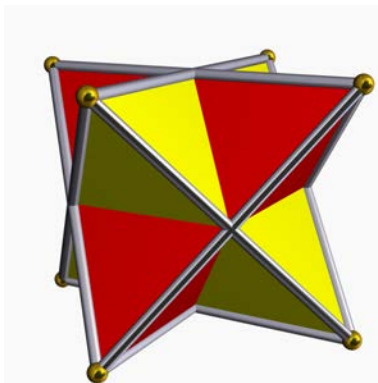
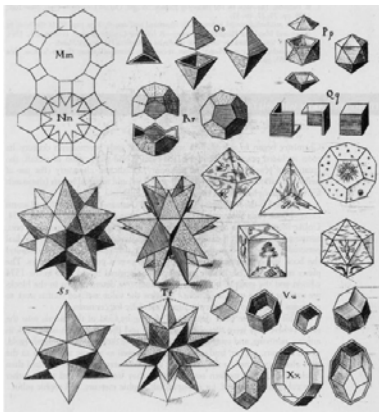




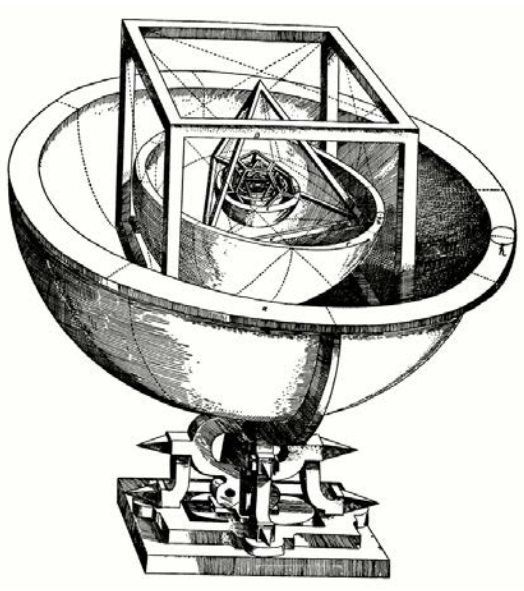


J. Kepler (1571–1630) *Mysterium Cosmographicum* (1596) y *Harmonices Mundi* II (1619)



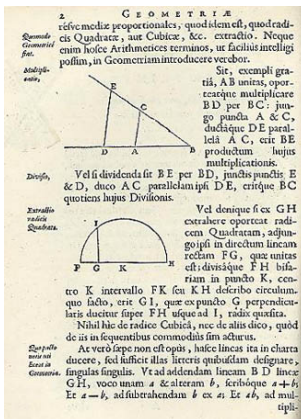


Poliedros estrellados, ¿Dualidad?



R. Descartes (1596–1650) *Progymnasmata de solidorum elementis*

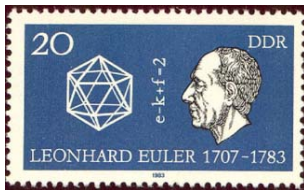




Teorema : la **suma** de los **defectos** en los vértices es 4π , para todo poliedro 3-dim

Defecto (o curvatura) en vértice V : lo que falta a la suma de ángulos faciales en V para llegar a $2\pi = 360^\circ$

Leonardo Euler (1707–1783)



Fórmula de Euler

$$c - a + v = 2 \quad (1750)$$

$$\xi(\mathbf{S}) = 2g - 2, \quad \xi(\mathbf{X}) = \mathbf{b}_0 - \mathbf{b}_1 + \mathbf{b}_2 - \mathbf{b}_3 + \cdots \quad (1895)$$



Característica Euler–Poincaré

Agustín Luis Cauchy (1789–1857)



Teorema de rigidez (1913) : si P, P' poliedros 3-dim con misma estructura combinatoria y con caras correspondientes congruentes $\Rightarrow P, P'$ congruentes.



Poliedros duales

✓ Tetraedro consigo mismo



✓ Cubo y octaedro



✓ Dodecaedro e icosaedro



duales de sólidos platónicos

David Hilbert (1879–1934) y Max Dehn (1878–1952)



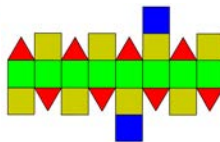
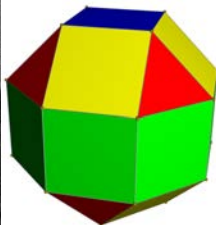
2–dim: Dado S polígono plano, cortamos S en una cantidad finita de polígonos y con los trozos armamos $T \Rightarrow \text{area}(S) = \text{area}(T)$ ¿Recíproco?
SI

Problema 3 de Hilbert :¿Se puede descomponer un tetraedro de vol 1 en una cantidad finita de poliedros y con ellos armar un cubo de vol 1? NO.

En 1901 Dehn introduce un invariante y calcula $D(\text{cubo}) = 0$,

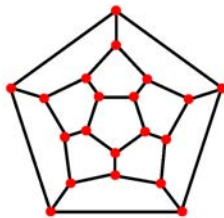
$D(\text{tetra}) \neq 0$

Ecuaciones de Dehn–Sommerville



Girobicúpula cuadrada elongada o J 37: es localmente regular por vértices, pero no transitivo en vértices.

Ernesto Steinitz (1871–1928) y Luis Schläfli (1814–1895)



- Steinitz: 1916 Caracterización **combinatoria** de poliedros convexos 3-dim
Teorema : Todo poliedro convexo forma un grafo plano 3-conexo y recíprocamente.
- Schläfli: en dim-4 hay 6 regulares; en dim superiores hay 3 regulares

Alejandro D. Aleksandrov (1912–1999)



- 1941 **Teorema de unicidad** : Si X espacio métrico geodésico homeomorfo a esfera y localmente euclídeo salvo en conjunto finito de puntos con defecto angular positivo y suma de defectos igual a 4π (rec. Descartes) $\Rightarrow X$ es desarrollo (rec. Durero) de un único poliedro convexo
- 1950 libro (en ruso) traducción alemana 1958, traducción inglesa 2005 Convex polyhedra

H.S.M. Coxeter (1907–2003) y Alicia Boole Stott (1860–1940)



- Coxeter: Años 70 varios libros : variantes concepto de regularidad, generalización a dimensión arbitraria, teoría poliedral de grupos (Klein), Convex Polytopes (1967): compendio, enfoque combinatorio
- Boole: en 4-dim, hay 6 politopos regulares



1947 Método del simplex, optimización, caminos sobre poliedros

Otros: : Richard Buckminster "Bucky" Fuller (1895–1983) (aplicaciones), N.W. Johnson, Branko Grünbaum (Convex polytopes, 1967) y Günter M. Ziegler (Lectures on Polytopes, 1995)



- ① St. Andrews Univ. History of Math.
<http://www-history.mcs.st-and.ac.uk/>
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- ⑨ P. Popescu-Pampu, La tante et les polyèdres, <http://images.math.cnrs.fr/La-tante-et-les-polyedres.html>
- ⑩ B. Grünbaum, Geometry strikes again, Math. Mag. **58**, n.1, 1985, 12–17