RADIUS AND DIAMETER OF WEAKLY OPEN SUBSETS IN BANACH SPACES

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In [3], Y. Ivakhno considered two different properties which, somehow, encode the fact that slices of the unit ball of a given Banach space are big. On the one hand, we say that a Banach space X has the *slice-D2P* if

$$d(X) := \inf\{\operatorname{diam}(S) : S \text{ is a slice of } B_X\} = 2.$$

On the other hand, we say that X has the r-BSP if, for every slice S of B_X we have,

$$r(S) := \inf\{r > 0 : S \subseteq B(x, r) \text{ for some } x \in X\} \geq 1.$$

Similarly to $d(X)$, we can define

$$r(X) := \inf\{r(S) : S \text{ is a slice of } B_X\}.$$

Y. Ivakhno proved in [3] that $d(X) = 2$ implies $r(X) \geq 1$. The converse was left as an open question. The question was completely solved in a negative way in [1]. Indeed, in [4] a Banach space X is constructed satisfying that $d(X) = r(X) = 1$.

The aim of this talk is to analyse whether it is possible to construct one such example replacing slices with weakly open sets. Namely, whether there is a Banach space X satisfying that the inf of the radius of non-empty relatively weakly open subsets of B_X is 1 and, at the same time, the inf of diameter of weakly open subsets is also 1.

Indeed, following results of [2], we construct a Banach space X where every non-empty weakly open subset of B_X has radius 1 and, at the same time, the inf of diameter of slices is also 1.

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