Characterizing sets where lip f is finite

Bruce Hanson

Given a continuous function $f : \mathbb{R} \to \mathbb{R}$, the upper and lower scaled oscillation functions are defined as follows:

$$\mathrm{Lip}f(x) = \limsup_{r \to 0^+} \frac{L_f(x,r)}{r},$$
$$\mathrm{lip}f(x) = \liminf_{r \to 0^+} \frac{L_f(x,r)}{r},$$

where

$$L_f(x,r) = \sup\{|f(x) - f(y)| \colon |x - y| \leq r\}.$$

We also define $L_f = \{x \in \mathbb{R} \,|\, \mathrm{Lip}f(x) < \infty\}$ and $l_f = \{x \in \mathbb{R} \,|\, \mathrm{lip}f(x) < \infty\}$. In my talk I will consider the problem of characterizing L_f and l_f. It is pretty straightforward to show that $S = L_f$ for some continuous function f if and only if S is an F_σ set. It is also not hard to show that every l_f is a $G_{\delta\sigma}$ set. I will consider the more difficult question of determining whether or not every $G_{\delta\sigma}$ set is equal to l_f for some f.