Characterizing sets where lip f is 0 and Lip f is infinite

Given $f : \mathbb{R} \to \mathbb{R}$ continuous, the Big and little lip functions are defined, respectively, as follows:

$$\operatorname{Lip} f(x) = \limsup_{r \to 0^+} M_f(x,r) \text{ and } \operatorname{lip} f(x) = \liminf_{r \to 0^+} M_f(x,r),$$

where

$$M_f(x,r) = \max_{|x-y| \leq r} \frac{|f(x) - f(y)|}{r}.$$

Then $\operatorname{Lip} f$ and $\operatorname{lip} f$ each measure the local Lipschtizness of f and, of course, are closely related. For example $f : \mathbb{R} \to \mathbb{R}$ is Lipschitz if and only if $\operatorname{Lip} f$ and $\operatorname{lip} f$ are bounded. On the other hand, these two functions can behave quite differently. For example, there exist $f : \mathbb{R} \to \mathbb{R}$ with $\operatorname{lip} f = 0$ a.e. and $\operatorname{Lip} f \equiv \infty$. In this talk, I will explore the question of characterizing various classes of *zero-infinity* sets, i.e. sets $E \subset \mathbb{R}$ such that there exists a continuous function $f : \mathbb{R} \to \mathbb{R}$ such that $\operatorname{lip} f(x) = 0$ and $\operatorname{Lip} f(x) = \infty$ on E.