SOME APPLICATIONS OF BOOLEAN ALGEBRAS AND STONE THEOREM TO FIXED POINT THEORY

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A metric space (B, d) is said to have the fixed point property (FPP) if every 1-Lipschitz operator $T : B \to B$ has a fixed point.

During this talk, our metric space will be the closed unit ball of a Banach space of continuous functions $C(K)$, for K a Hausdorff compact topological space. We are interested in identifying topological properties of the compact set K that are connected to the failure or to the fulfilment of the FPP for the closed unit ball B of $C(K)$.

This question arises after observing that the two opposite behaviours hold: the closed unit ball of $C(K)$ fails the FPP when K is the one-point compactification of $\mathbb{N}$. In contrast, the closed unit ball of $C(K)$ does verify the FPP when $K = \beta\mathbb{N}$, the Stone-Cech compactification of $\mathbb{N}$ (extremally disconnection, hyperconvexity and injectivity play their role here). For the remainder, $\mathbb{N}^* := \beta\mathbb{N} \setminus \mathbb{N}$, it is not known whether the closed unit ball of $C(\mathbb{N}^*)$ has the FPP.

While $C(\beta\mathbb{N})$ is isometric to the sequence space ℓ_∞, $C(\mathbb{N}^*)$ is isometric to the quotient Banach space ℓ_∞/c_0. A natural question arises: Does the closed unit ball of ℓ_∞/c_0 have the FPP?

Assuming the Continuum Hypothesis (CH) and with the aid of Boolean algebras, we will prove that the closed unit ball of ℓ_∞/c_0 fails the FPP.

We will raise the question of characterizing all compact sets K for which the FPP holds for the unit ball of $C(K)$ and we will expose some partial and interesting results.

The results included in this talk will be eventually published in a joint paper together with

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https://arxiv.org/pdf/2506.17995