CONSTRUCTION OF QUASICOMPLEMENTS WITH A PRESCRIBED BEHAVIOR

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In this talk we acquaint with problems involving quasicomplements and operator ranges in Banach spaces, providing several recent results.

First, we show that if X and Y are quasicomplements but not complements in a Banach space E and X has a separable quotient, then Y admits a quasicomplement $X_1 \subset X$ such that $\dim(X/X_1) = \infty$. We also prove that if X has a weak*-separable dual, then for each proper dense operator range R in X, the subspace X_1 can be choosen so that $R \cap X_1 = \{0\}$.

Next, we prove that for a Banach space E with a separable quotient, E^* is weak*-separable if and only if for every closed subspace $X \subset E$ and every proper dense operator range R in E containing X there exists a quasicomplement Y of X such that $R \cap Y = \{0\}$. Finally, we show that if a Banach space E has separable quotient, then every proper dense operator range R in E is covered by the sum of two quasicomplementary non-complementary subspaces of E.

The talk is based on a joint work with Mar Jiménez Sevilla and Sebastián Lajara.