CLOSED EXTENSION SETS FOR BV AND SOBOLEV SPACES

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The study of extendability of different function spaces from subsets of Euclidean space has a long history. Classical questions include existence of bounded extension operators from function spaces on domains to the function spaces on the whole space and the extendability of functions with prescribed values (and derivatives) in (arbitrary) sets, such as C^k extension from a finite set of points or biLipschitz extensions from the boundary of a domain.

In this talk I will focus on the properties of closed sets of positive measure that have a (not necessarily linear) extension operator from the BV-space defined on the closed set to the BV-space defined in the ambient space. Here the underlying space that we consider are general metric spaces, PI-spaces and the Euclidean plane. The study of closed BV-extension sets is motivated by two observations. Firstly, in general metric measure spaces we have been able to prove that there exist many closed BV-extension sets, while for open sets this is unknown. Secondly, for sets with measure zero topological boundary, the closure of the set typically inherits the extension properties that the original set had, but this conclusion cannot be reversed.

Since the closure of extension sets inherit the extension properties of the original set, it is natural to ask if the closure actually improves these properties. The specific question I have considered with Emanuele Caputo, Jesse Koivu, and Danka Lučić (`https://doi.org/10.1016/j.jfa.2025.111319`) is if closed BV-extension sets are actually $W^{1,1}$-extension sets. Our preliminary result is that although this is not true in general metric spaces, it does hold for finitely connected planar sets.