Level sets of prevalent Weierstrass functions

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The α-Weierstrass function is defined as $W_g^{\alpha,b}(x) = \sum_{k=0}^{\infty} b^{-\alpha k} g(b^k x)$, where g is a Lipschitz function on the unit circle. For a prevalent α-Weierstrass function, we prove that the upper Minkowski dimension of every level set is at most $1 - \alpha$, and the Hausdorff dimension of almost every level set equals $1 - \alpha$ with respect to its occupation measure. We further demonstrate that the occupation measure of a prevalent α-Weierstrass function is absolutely continuous with respect to the Lebesgue measure. Consequently, the result on the Hausdorff dimension of level sets applies to a set of level sets with positive Lebesgue measure. A central tool in our analysis is the Weierstrass embedding. For a sufficiently large dimension d, we construct Lipschitz functions $g_0, g_1, \ldots, g_{d-1}$ such that the mapping $x \mapsto \left(W_{g_0}^{\alpha,b}(x), W_{g_1}^{\alpha,b}(x), \ldots, W_{g_{d-1}}^{\alpha,b}(x)\right)$ is α-bi-Hölder. We also prove that such an embedding requires at least $1/\alpha$ coordinate functions.

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