# Two views in Numerical Astronomy: Predictability in Galactic Potentials Evolution of Protoplanetary discs

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# Forecast

- Forecasting is the **process of making predictions** of the future based on past and present data.

- One of the key aspects of **the scientific method** is the possibility of making predictions using the selected model and to confront these predictions with new observations.

- A dynamical model is typically described by differential equations, which are obtained by the analysis of the physical system at a fundamental level, yet **involving approximations** sufficient to simplify the model.

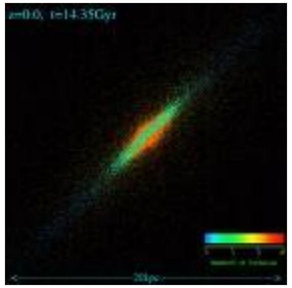
# Numerical Astronomy

- Astronomy is an **experimental** science.
- Astronomers have no direct means to **gain access to the observed** objects (in general).
- Astronomers can not **modify the main parameters** of the studied systems.
- Evolutionary **time scales** of astronomical systems are totally different from common human time scales.

- Therefore, **numerical simulations** are a key tool for any analysis.

$$\partial_t f + v\nabla_x f - \nabla_x \psi \nabla_v f = 0$$

$$\psi(x,t) = -4\pi \int \frac{f(x',v',t)}{|x-x'|} d\zeta'$$



# Reliability

- With the **widespread use of computer simulations**, the reliability of numerical calculations is of increasing interest.

- This reliability is related to the **instability properties** of:
  - The analysed orbits (local) and model (global).
  - The numerical schemes.

- Modelling is an **interdisciplinary** field:
  - Astrophysics provides the simulated models.
  - Theoretical physics provides the instability properties.
  - Computational sciences provide the numerical implementation.

# Context

- If the system to be solved is **only known approximately**, it is meaningless to try to solve it with great accuracy.
- It can be enough **to solve the approximate system** and to focus on assuring that the numerical resolution will not introduce large errors, keeping them, at least, of the same order of magnitude than the error introduced by having an approximated model.
- We should know at least what are the **implications** of their existence for the intended forecast.

- This talk presents two different approaches within this context.

# Predictability in Galactic Potentials

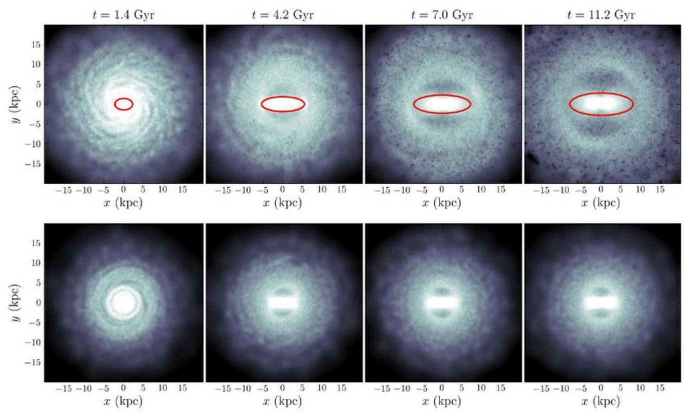
Universidad
Rey Juan Carlos

# Deterministic Equations

- The gravitational N-body simulation is a common tool for studying galactic systems.

- An alternative is the use of simulations based on **a single mean field potential**.
  - Some potentials are specific snapshots of the N-body simulations.
  - Some potentials are selected to physically represent desired characteristics of the galaxies.

- Fully deterministic equations, but some exhibit a **strong sensitivity on the initial conditions**.



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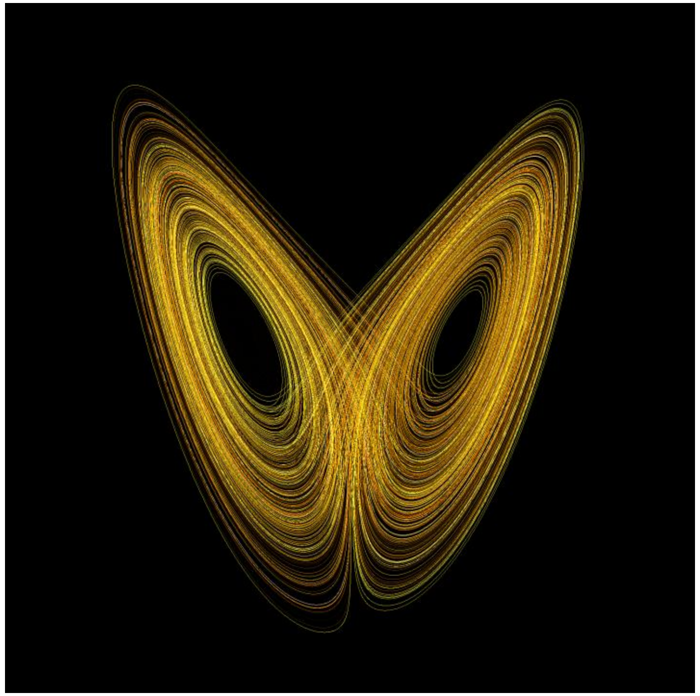
# Solving ODEs

- ODE and predictions: we need to **know initial condition**.
- The physical systems showing strong sensitivity are called “chaotic”.
- American meteorologist Edward Lorenz published his article “Deterministic Non- periodic flow” in 1963. Working with a computer, he found and presented many of the ideas that today belong to the **chaos theory**.



# Chaos and Predictability

- A system is said to be **chaotic** when the exact solution and a numerical solution starting very close to it may diverge exponentially one from each other.

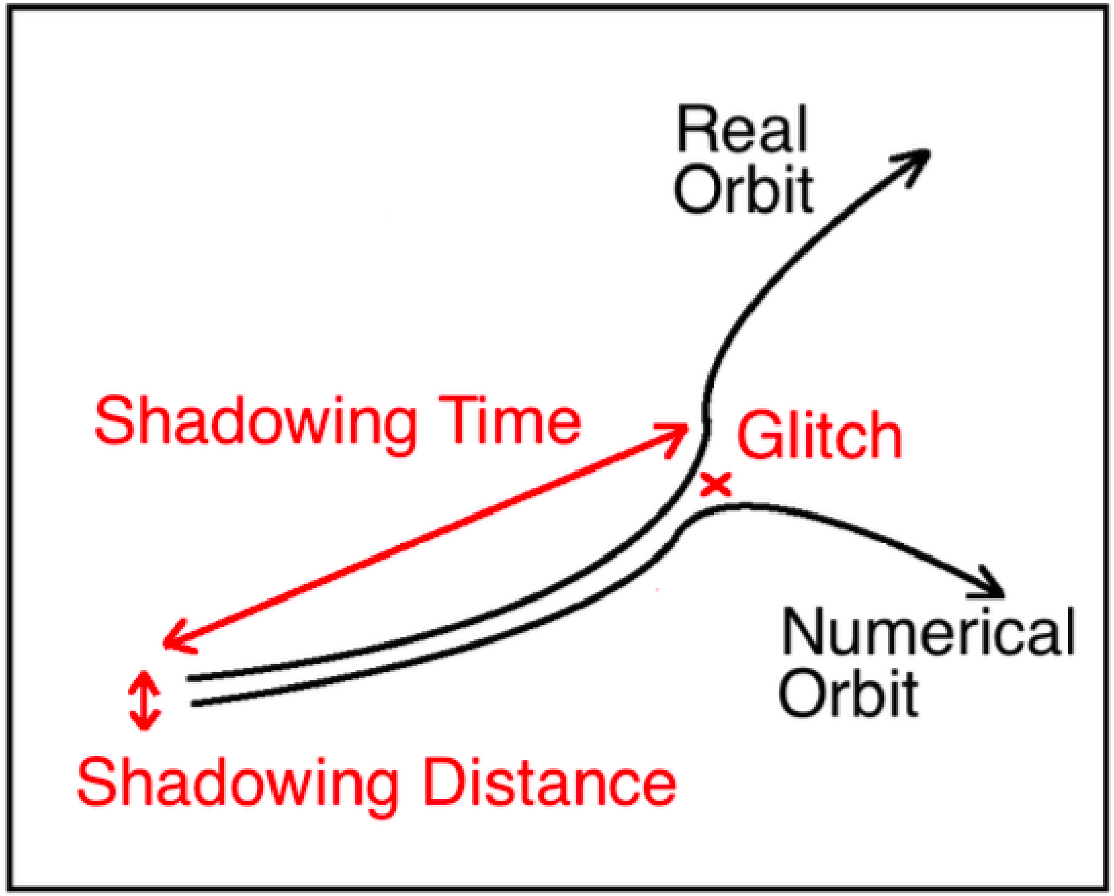
- The **predictability** of a system indicates how long a computed orbit is close to an actual orbit.

- The predictability aims to characterise if this numerically computed orbit may be still reflecting real properties of the model, leading to correct predictions.

- The real orbit is called a **shadow**, and the noisy solution can be considered an experimental observation of one exact trajectory.

# Shadowing

- Consequently, a proper estimation of the **shadowing times** is a key issue in any simulation, and provides and indication about its predictability.



# Shadowing (II)

- The **shadowing distance** can be described as a diffusion process, or biased random walk with a drift towards a reflecting barrier.

- When we use the closest to zero exponent, and assume mean (m) and standard deviation (σ) of **finite-time Lyapunov** exponents to be very small, the **shadowing time** is,

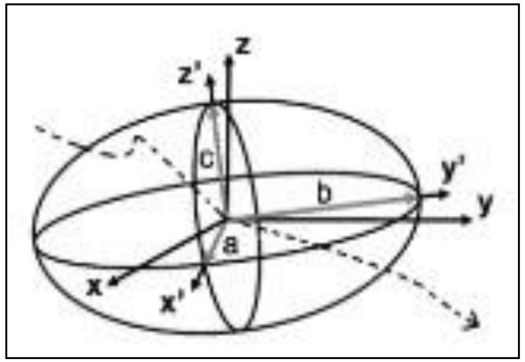
$$\tau \sim \delta^{-h} \qquad h = \frac{2\|m\|}{\sigma^2}$$

Sauer T., Grebogi C. and Yorke J., Physics Letters A, 79, 59 (1997)
Sauer T., Physical Review E, 65, 036220 (2002)

# Lyapunov Exponents

- The ordinary, or **asymptotic, Lyapunov exponent** λ describes the evolution in time of the distance between two nearly initial conditions, by averaging the exponential rate of divergence of two trajectories starting from a given deviation vector.
- The **finite-time Lyapunov exponents** derive from above for finite averaging times.

$$\lambda = \lim_{t \to \infty} \lim_{\delta z(0) \to 0} \frac{1}{t} \ln \frac{\delta z(t)}{\delta z(0)}.$$



# Exponents Distributions

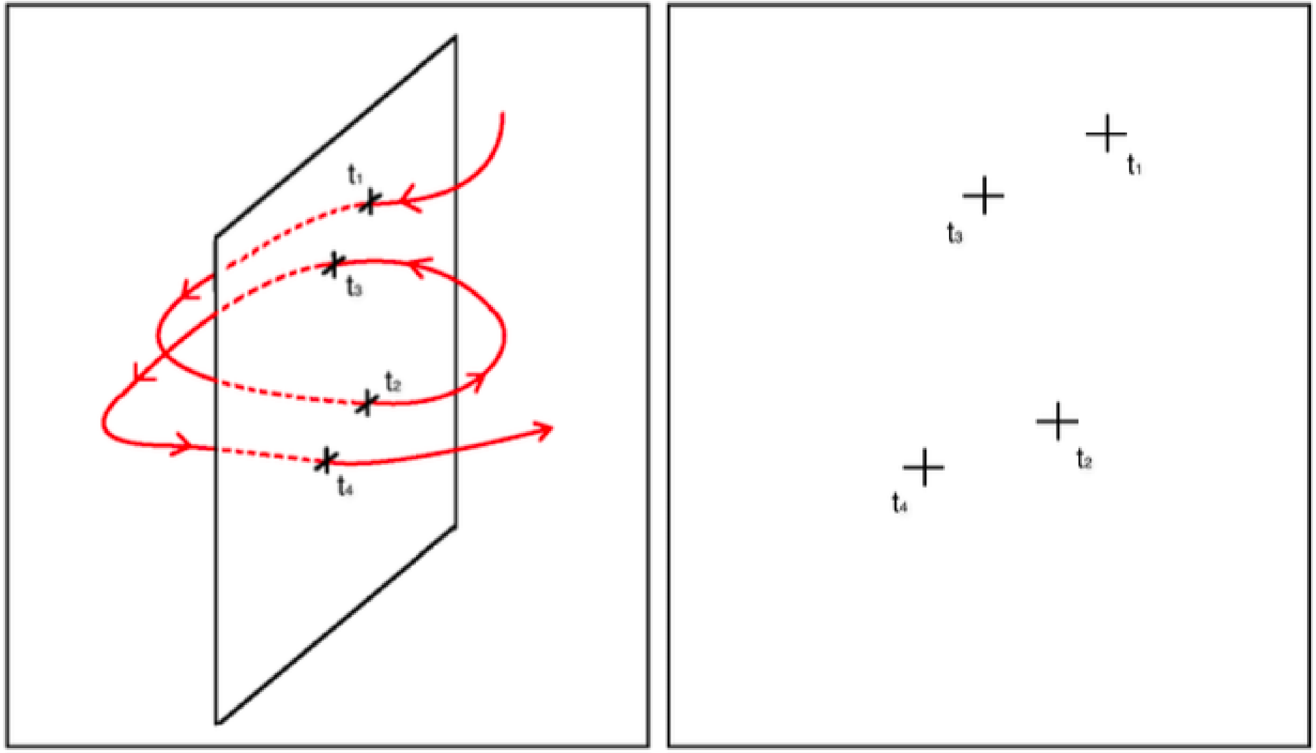
- A sign of **bad shadowing** is the fluctuating behavior around zero of the finite-time Lyapunov exponent closest to zero.

- These distributions,
  - Depend on **the initial orientation**.
  - Depends on the **total integration time**. We can or can not analyse transients dynamics.
  - Depend on the **finite-time interval length**.

- We fix an arbitrary initial set of deviation vectors allowing them to evolve under the flow dynamics.

# Poincaré Sections

- Because the time-scales are important, we analyse the role of Poincaré **averaged crossing time**.
- Poincaré sections are a very well known tool for analysing dynamical systems.

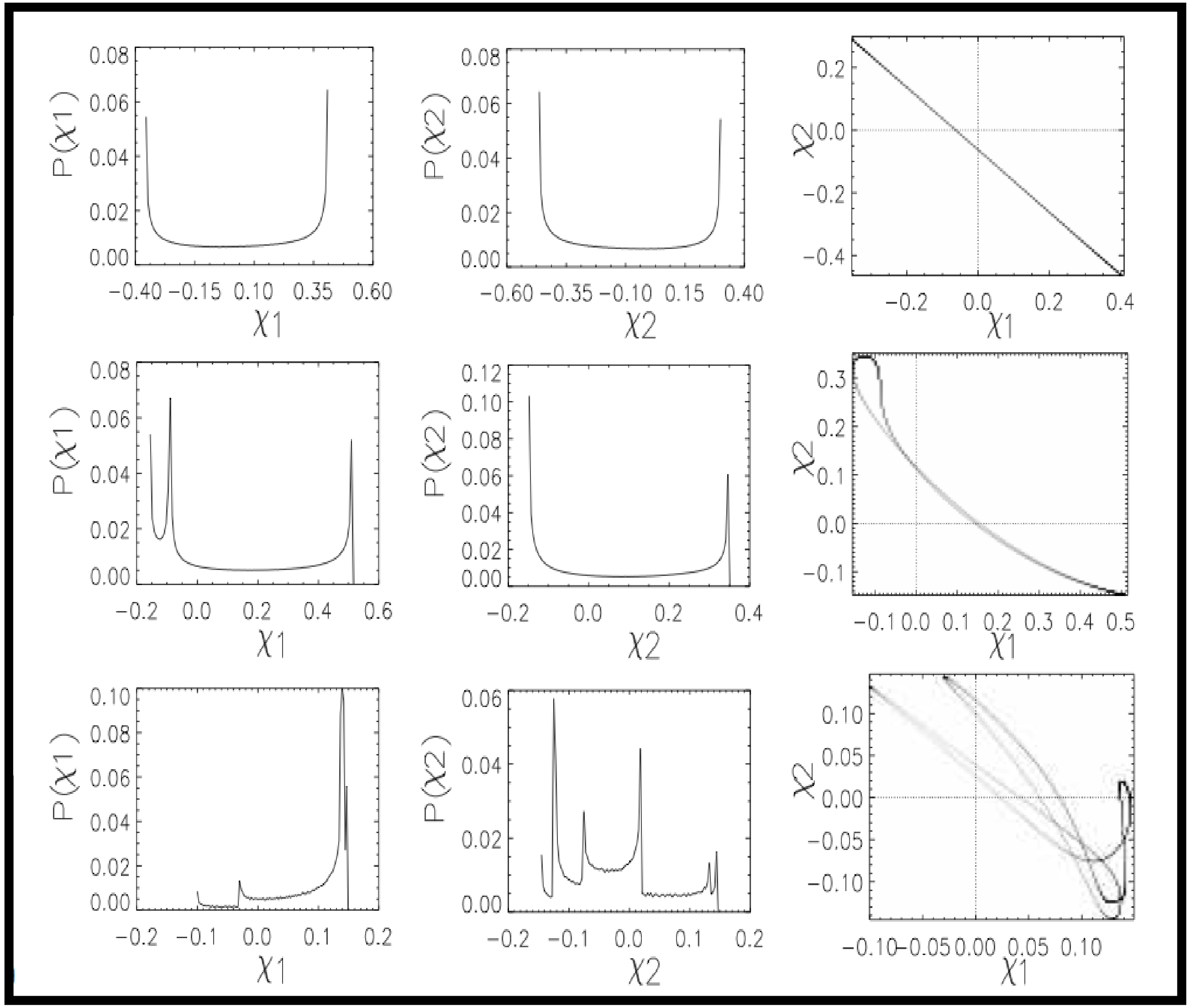


# Exponents Distributions

Δt= 0.01

Δt= 1.0

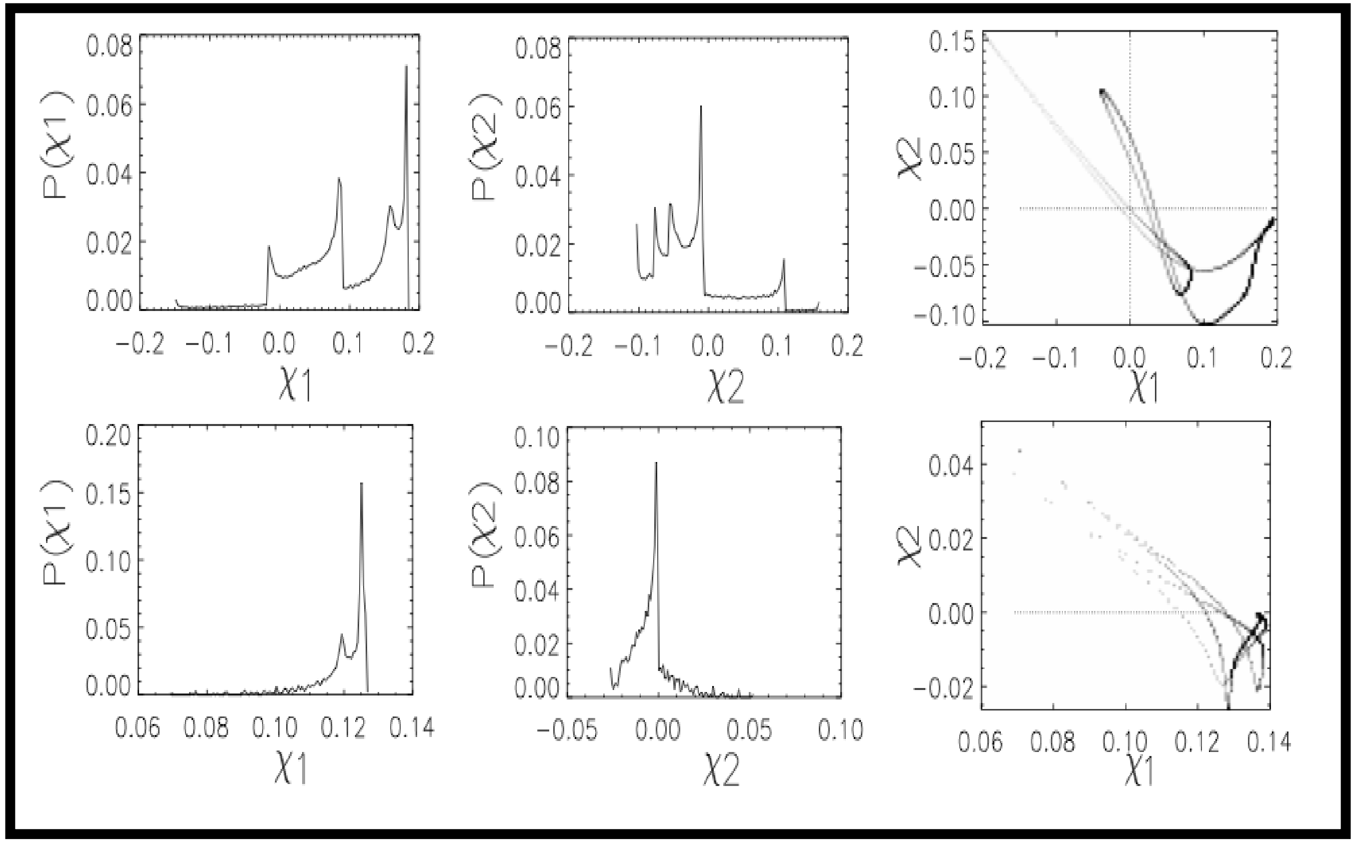
Δt= 7.0



# Exponents Distributions

Δt= 10.0

Δt= 100.0




- When reaching the Poincaré cross-section time scale, the system leaves the local regime and enters in the **global regime**.

**Vallejo J.C, Viana R. Sanjuan M.A.F, Physical Review E, 78, 066204 (2008)**



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# Numerical Technique

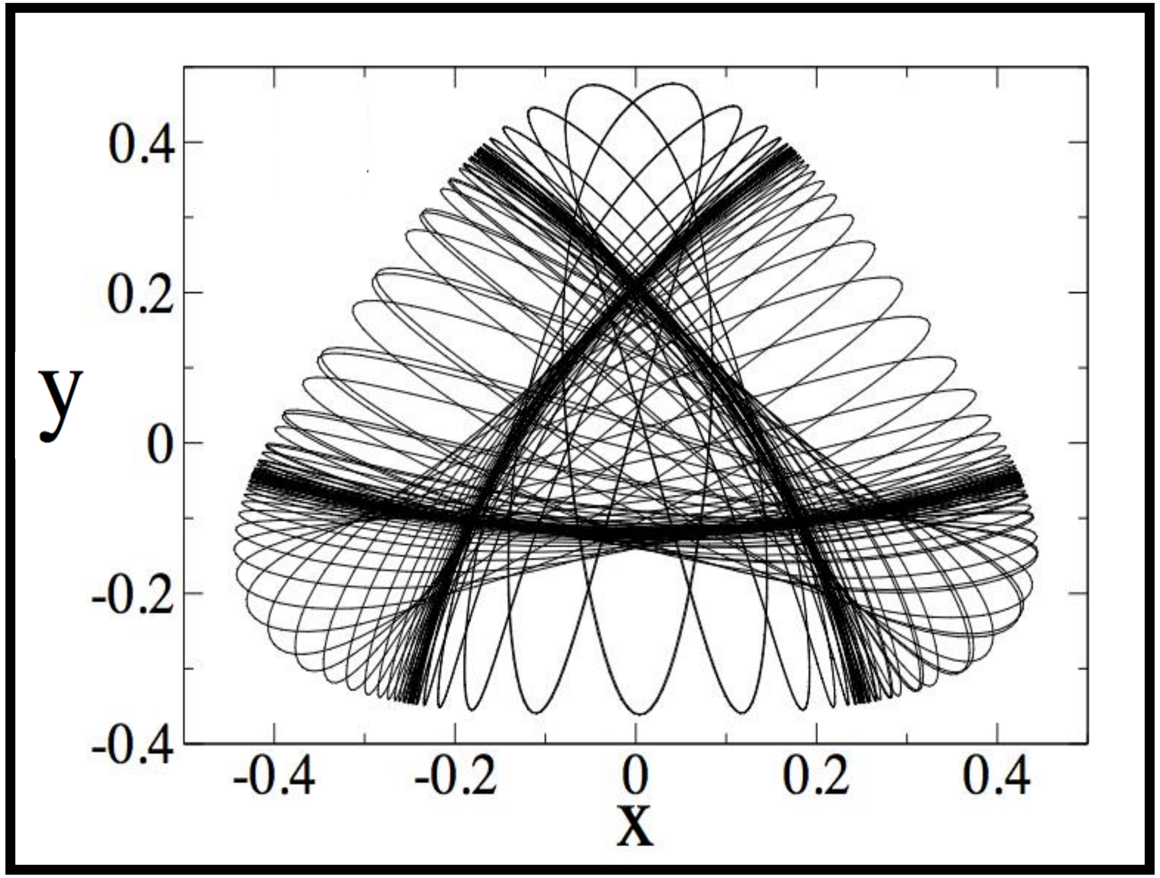
- In a general system every orbit has its own dynamics and time scales. There are strong dependencies on the finite-time length.

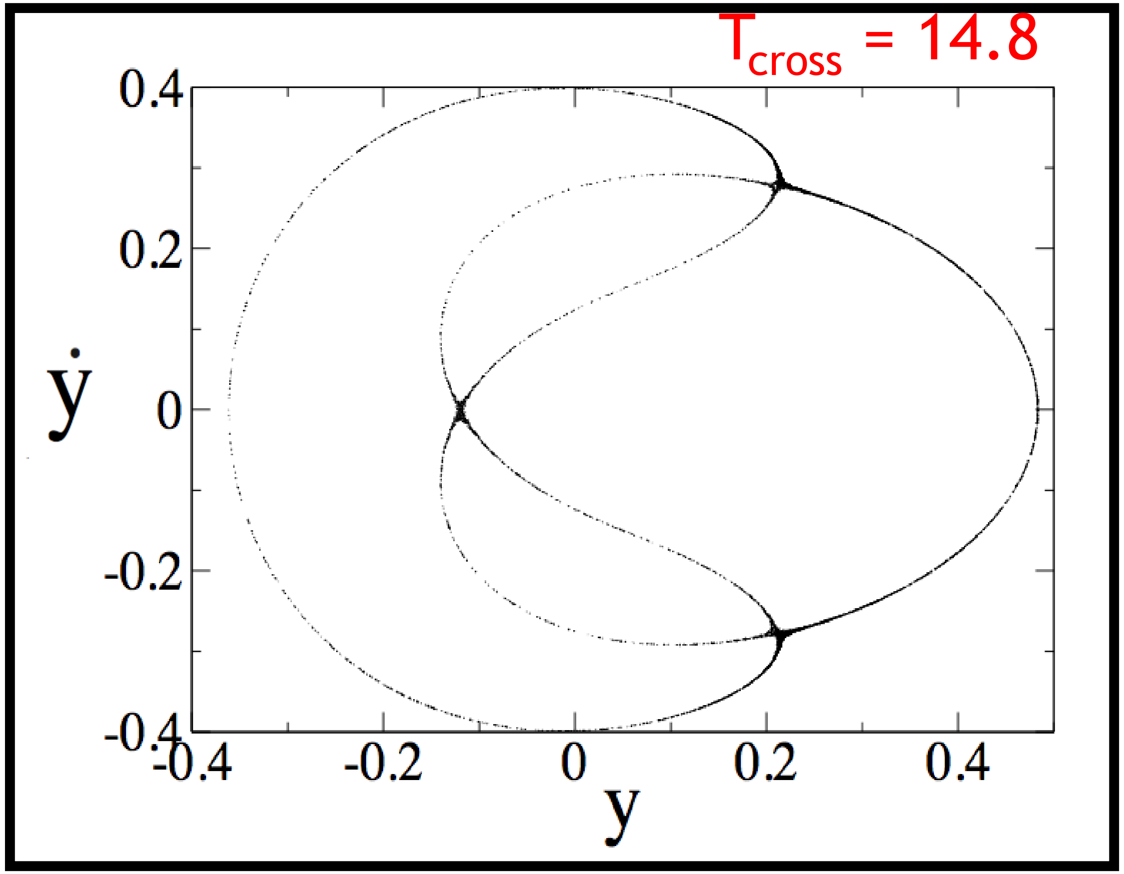
- Use the **kurtosis** for getting insight into the interval size where the global regime is reached.

- **Cross-check** this interval length with the Poincaré cross-section time.

- We have applied these ideas to simple meridional potential systems (Hénon-Heiles, Contopoulos) and a Galaxy model of three components.

# Hénon-Heiles



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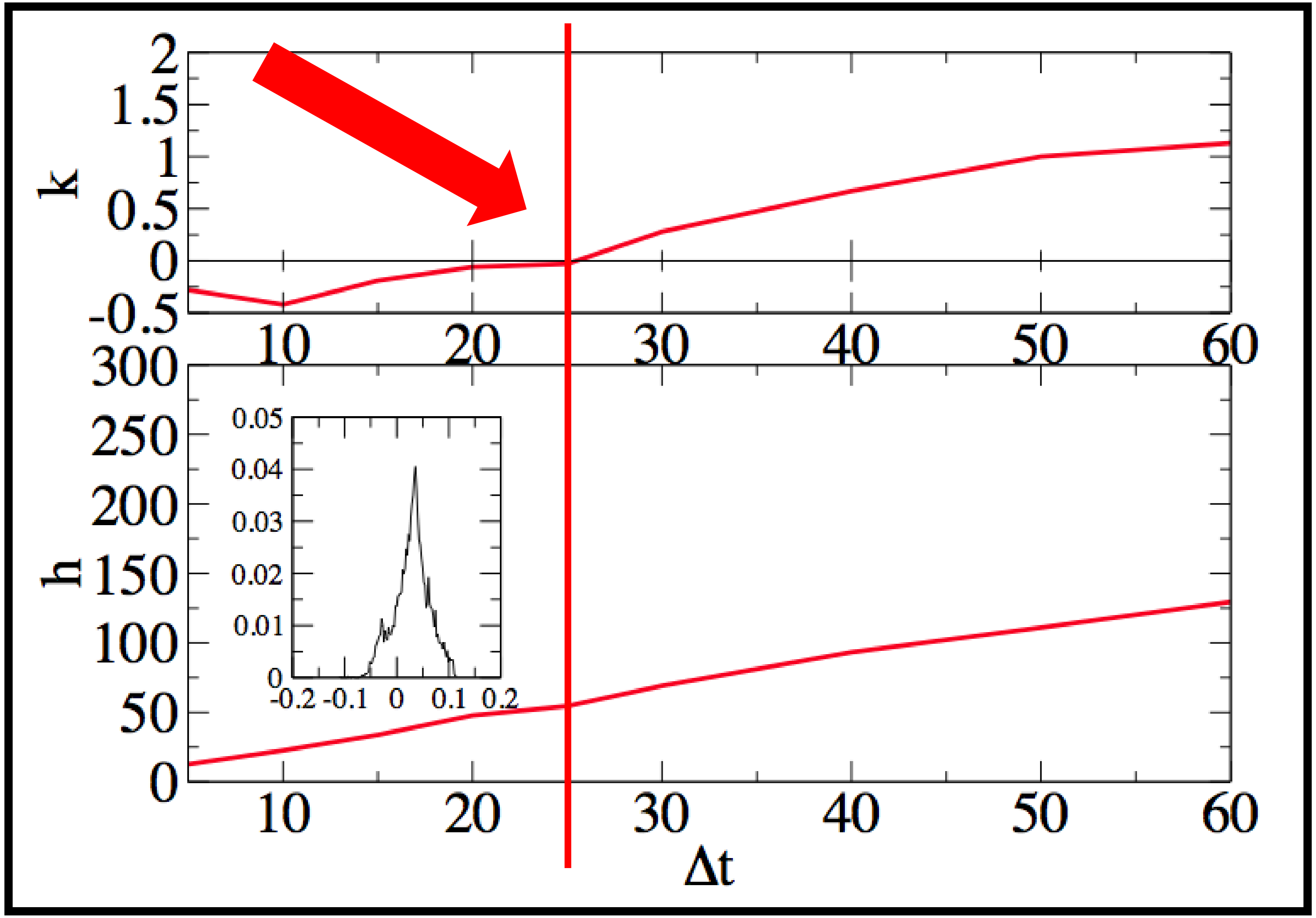
# Predictability



# Galactic Potential

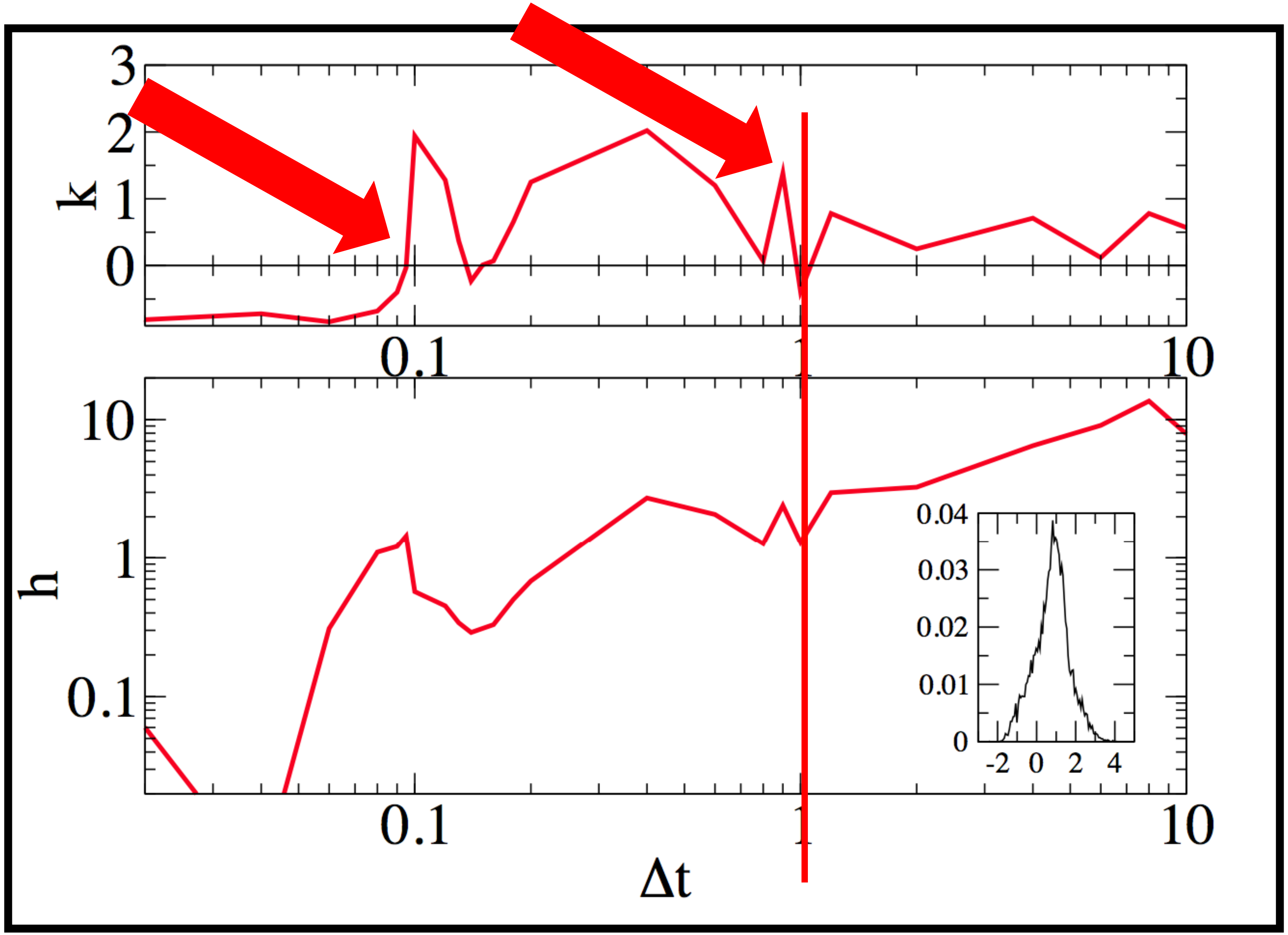
$$V = \Phi_{disk} + \Phi_{sphere} + \Phi_{halo}$$

$$\Phi_{\rm disk} = -\alpha \frac{GM_{\rm disk}}{\sqrt{R^2 + (a + \sqrt{z^2 + b^2})^2}},$$

$$\Phi_{\rm sphere} = -\alpha \frac{GM_{\rm sphere}}{r + c},$$

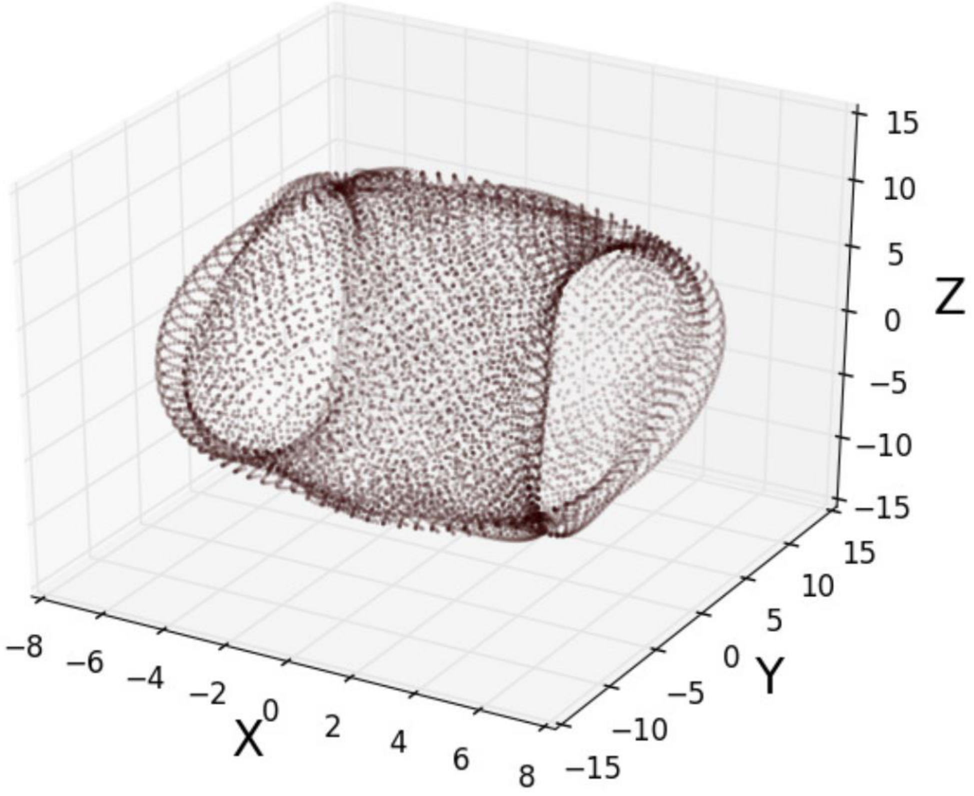
$$\Phi_{\rm halo} = v_{\rm halo}^2 \ln (C_1 x^2 + C_2 y^2 + C_3 xy + (z/q_z)^2 + r_{\rm halo}^2),$$

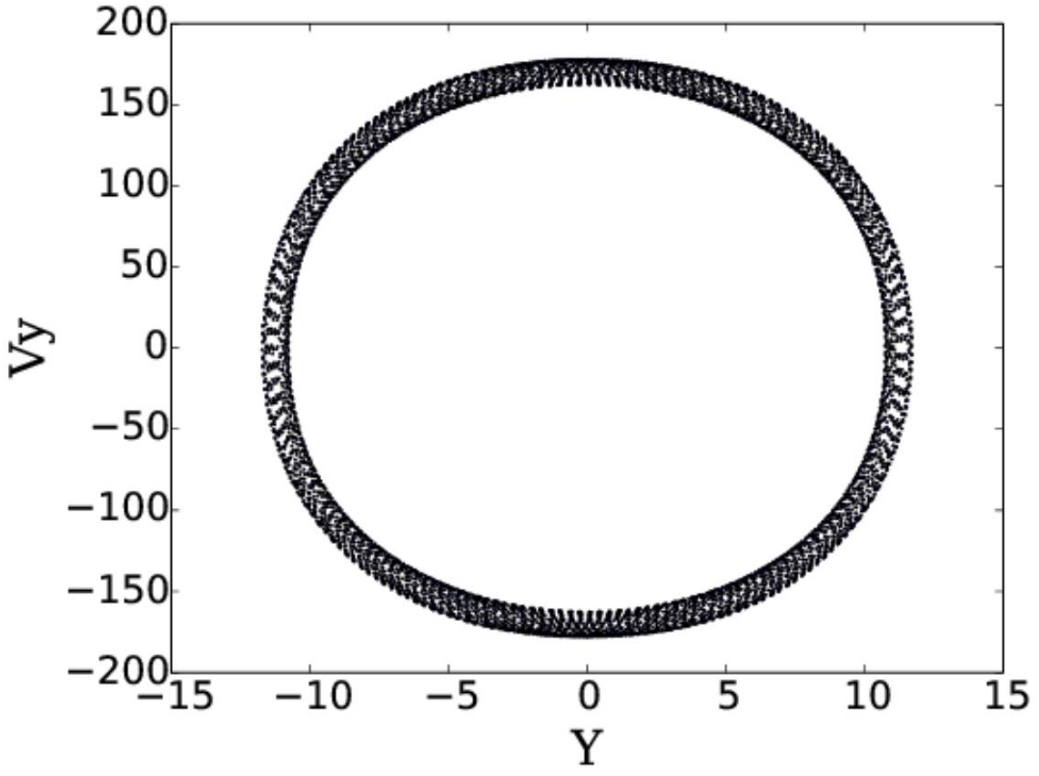
# Predictability



# Role of Halo Orientation

HC2

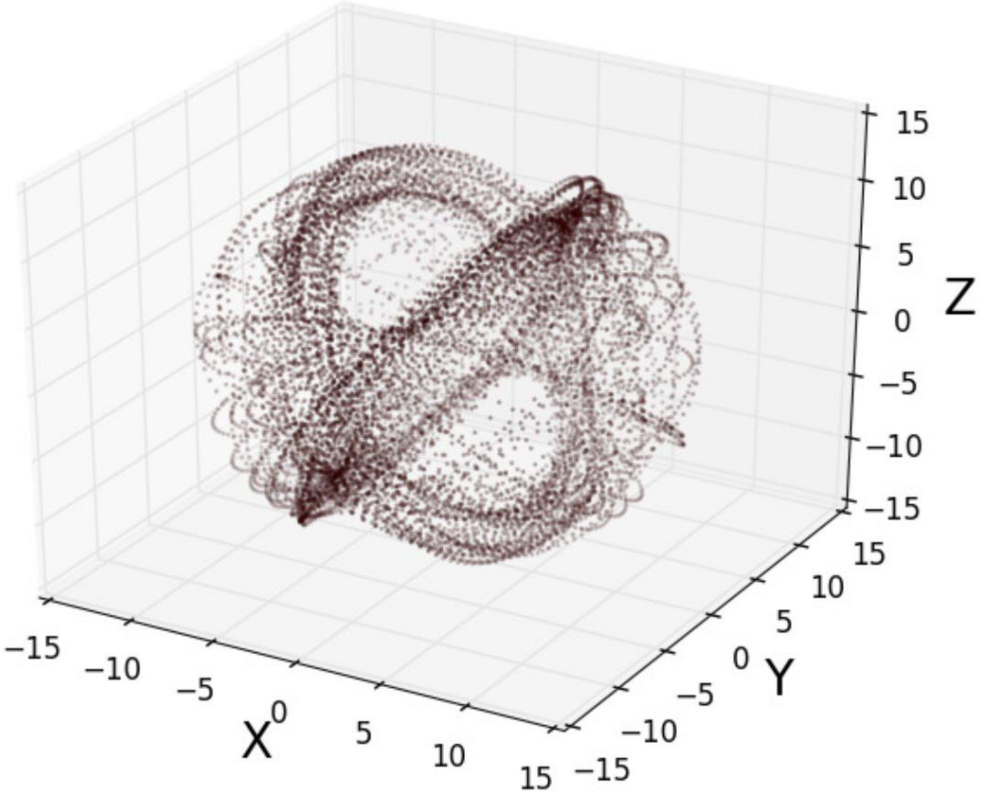


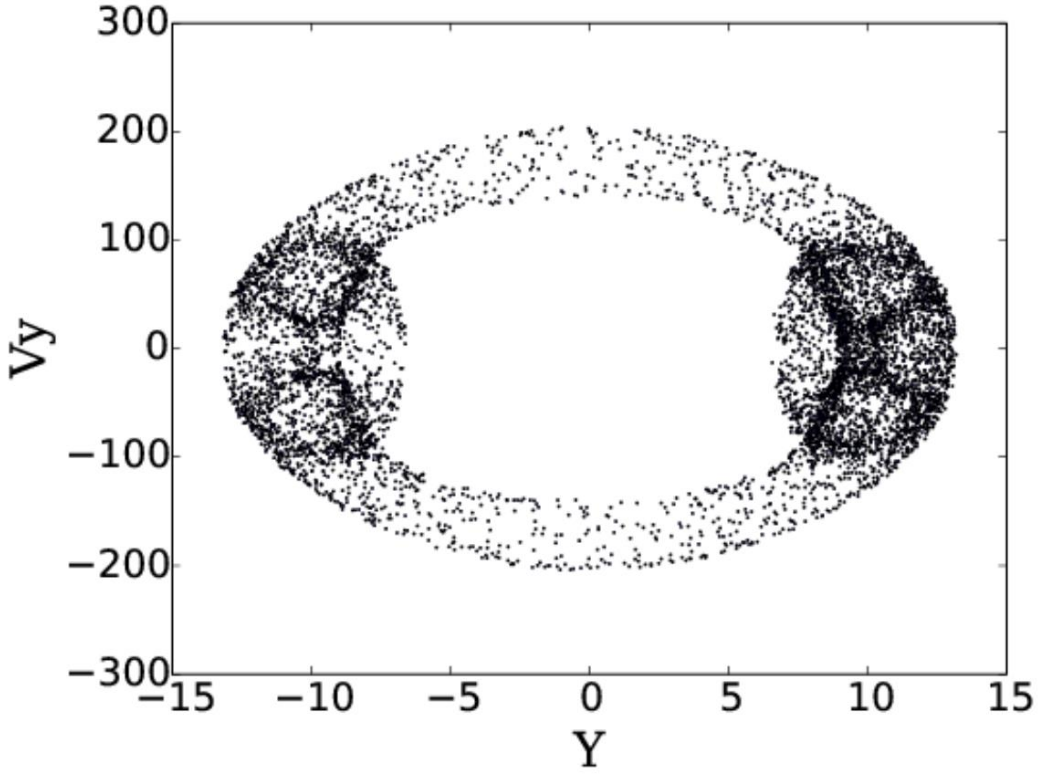




$$\lambda = 0.00 \; h = 0.41$$

# Role of Halo Orientation

HC2




$$\lambda = 0.43 \; h = 0.03$$



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# Some Results

- The variation of the shapes of the finite time Lyapunov exponents allows to select **the most adequate** interval for having a reliable predictability index.

- When one calculates the **predictability index** in a variety of orbits and models, some results are:
  - Chaoticity is not the same that low predictability.
  - Not all regular orbits have the same predictability.
  - Identification of orbits with very low predictability.

- Sometimes, a **sophisticated** numerical scheme may be needed. Conversely, in other cases, the **limited gain in reliability** does not compensate the sophistication.

# Some Results

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY
MNRAS **447**, 3797–3811 (2015) doi:10.1093/mnras/stu27

## The forecast of predictability for computed orbits in galactic models

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Astronomy & Astrophysics

## Role of dark matter haloes on the predictability of computed orbits

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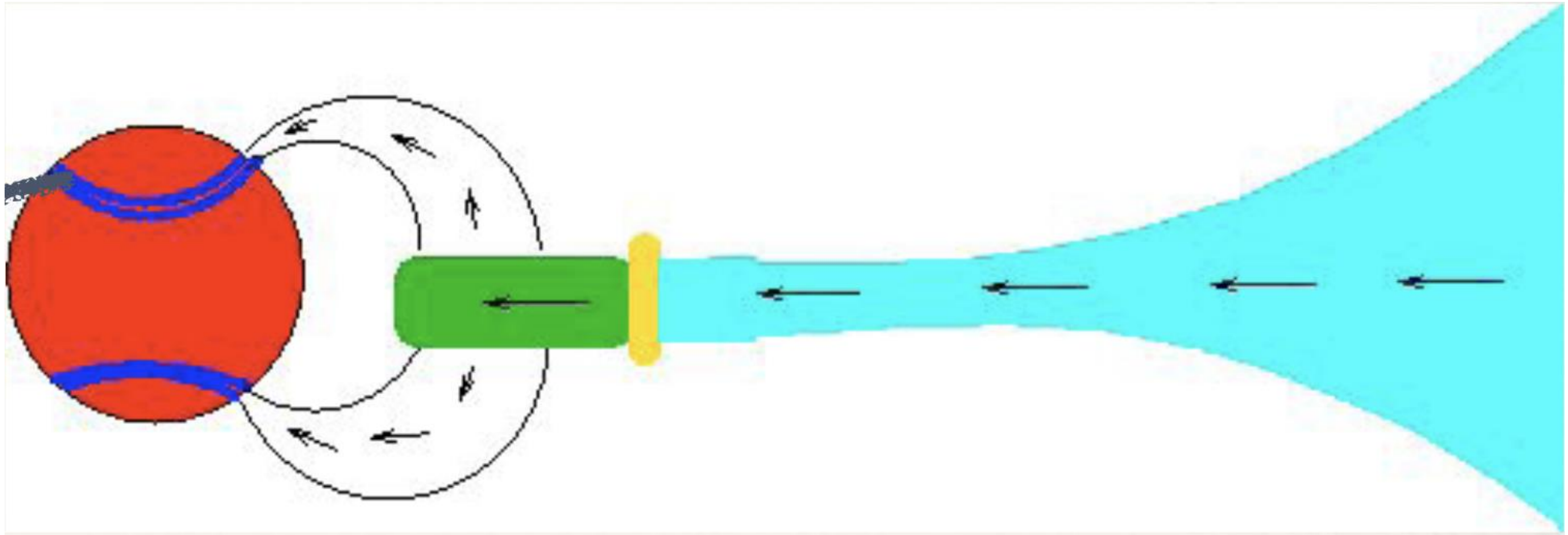
# Evolution of protoplanetary discs

# Context

- Two fundamental sources of error in any simulation: **truncation** error and **discretisation**.
- We have (roughly) discussed the first.
- The **numerical scheme** solves a problem that is not exactly the same we aim to solve.
- The approximate solution obtained with one numerical scheme can be viewed as the exact solution of a different problem.

# Accreting discs

- Evidence for **accretion** in protoplanetary discs.
- In order to accrete onto the star, the **angular momentum** from the gas must be lost or redistributed in the disc.
  - **Redistributed** within the disc via a viscous process.
  - **Lost** from the star-disc system via outflows.



# The model

- The evolution of an accretion disc is regulated by two conservation laws: conservation of mass, and conservation of angular momentum.
- We assume a simple power law in radius for viscosity.
- We obtain **a diffusion equation** for the evolution of the surface density of a thin disc (with proper change of variables)

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial}{\partial r}\left(\nu \Sigma r^{1/2}\right)\right]$$

$$\frac{\partial f}{\partial t} = D\frac{\partial^2 f}{\partial X^2}$$



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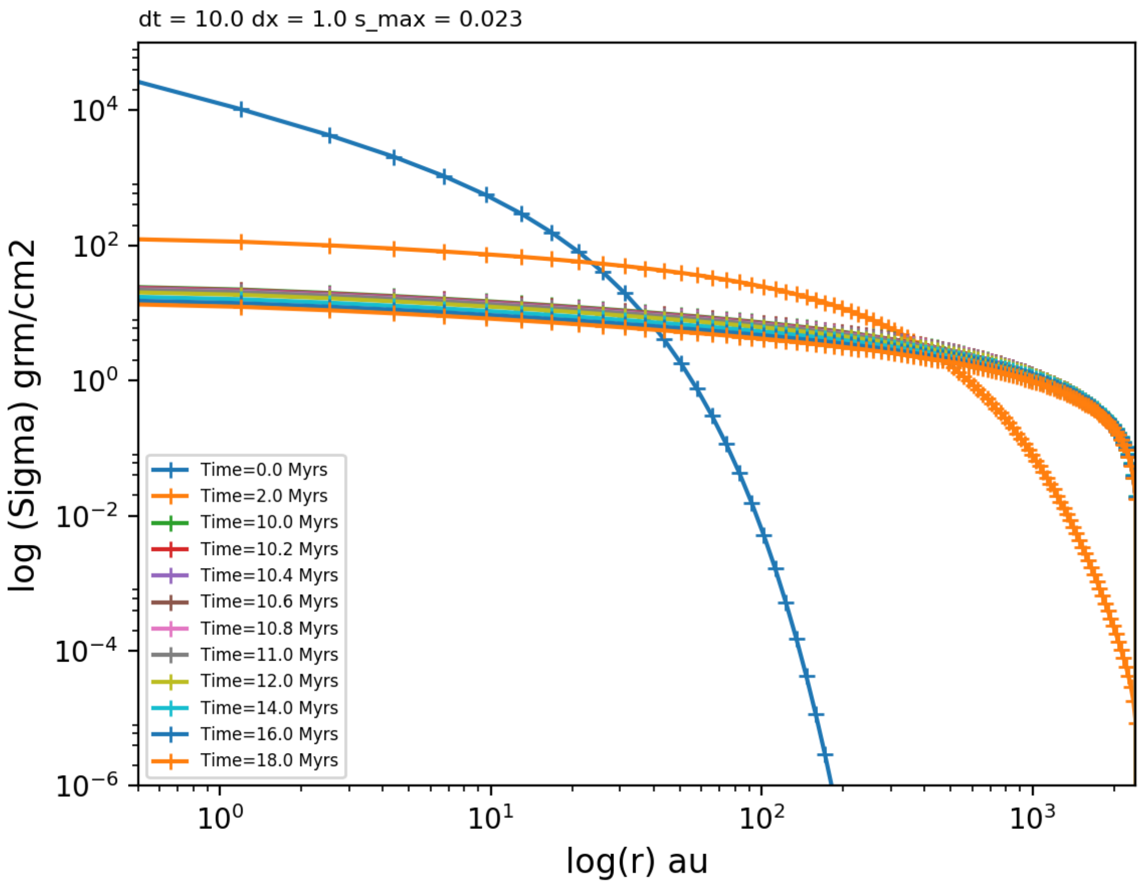
# Simple viscous disc evolution

- Over time the surface mass density decreases while the size increases (to conserve angular momentum). But only one time scale is seen.



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# UV-switch model

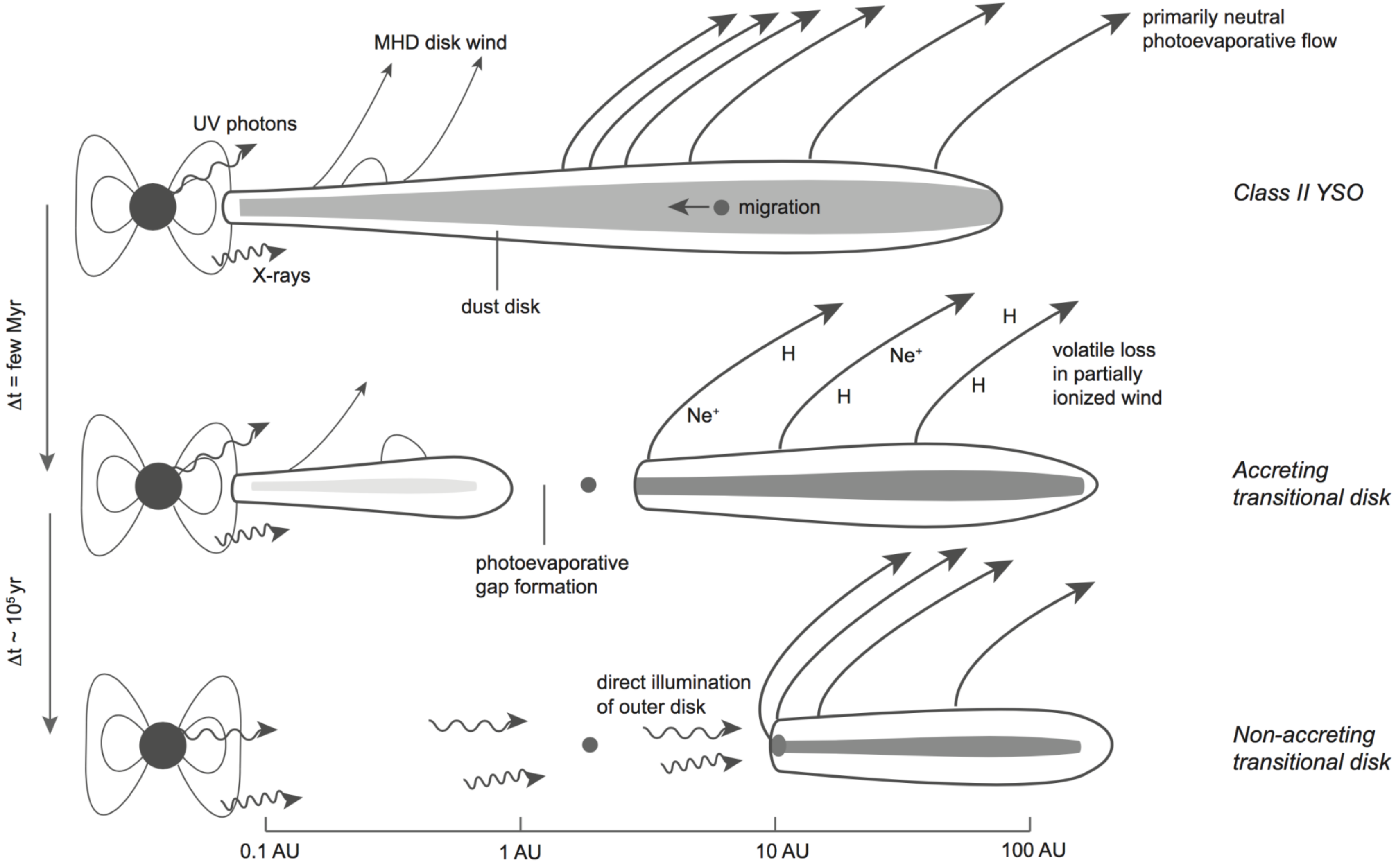
- **Time scales** from transition between disc-possesing and discless status seems to be smaller.

- **Additional mechanisms** invoked:
    - Stellar magnetospheric field clear out.
    - Planet formation.
    - Photoionization by central star.

# (Internal) Photoevaporation

- Irradiation of disk surface creates a thin surface layer of hot gas.
- If its temperature/thermal velocity exceeds the local escape velocity, the surface layer gets unbound and evaporates, i.e. a thermal wind is launched taking away disk gas.
- We follow **EUV driven evaporation** as modeled by Clarke, Gendrin & Sotomayor 2001.

# (Internal) Photoevaporation




From Alexander et al, arXiV:1311.1819 (2013)

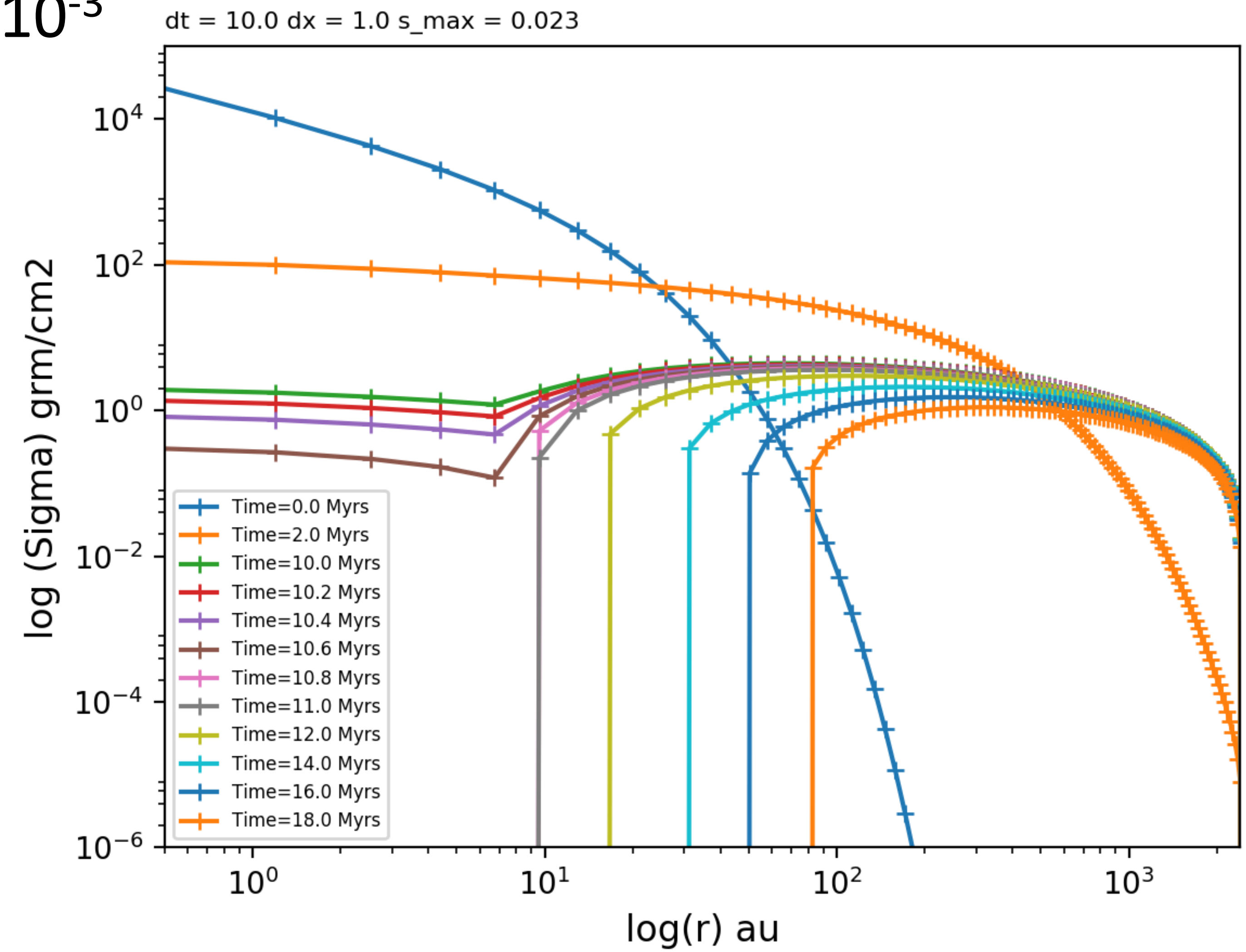
# Simple model

- If the disc losses mass due to photoevaporation and if the gas has the same specific angular momentum as the disc:

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial}{\partial r}\left(\nu \Sigma r^{1/2}\right)\right] + \dot{\Sigma}$$

# Simple model (II)

- $r_{min}$=9.0 $10^{-3}$



# Reproducible Science

The viscous evolution of the disc subject to photoevaporative mass loss is computed using a standard first-order explicit scheme, equispaced in $R^{1/2}$ over the dynamic range $9 \times 10^{-3}$–$250 R_1$. Zero-torque boundary conditions are applied at the inner and outer boundaries. For ease of comparison with HCGA, the spectral

This fixes the scaling constant $\nu_0$, and thus the disc model is entirely specified by the three parameters $M_d(0)$, $\dot{M}_d(0, 0)$ and $R_0$. Initially, we adopt $M_d(0) = 0.05\,M_\odot$, $\dot{M}_d(0, 0) = 5.0 \times 10^{-7}\,M_\odot\,yr^{-1}$ and $R_0 = 10$ au. We use 1000 grid points, equispaced in $R^{1/2}$, which span the radial range [0.0025, 2500 au]. We also adopt zero-torque boundary conditions throughout (i.e. we set $\Sigma = 0$ at the grid boundaries) but note that the spatial domain is always large enough that the outer boundary condition has no effect on the results.



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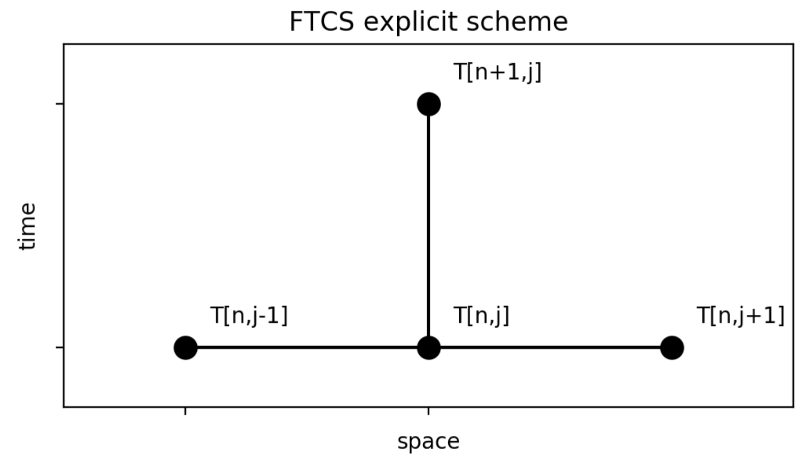
# PDEs and Boundary Conditions

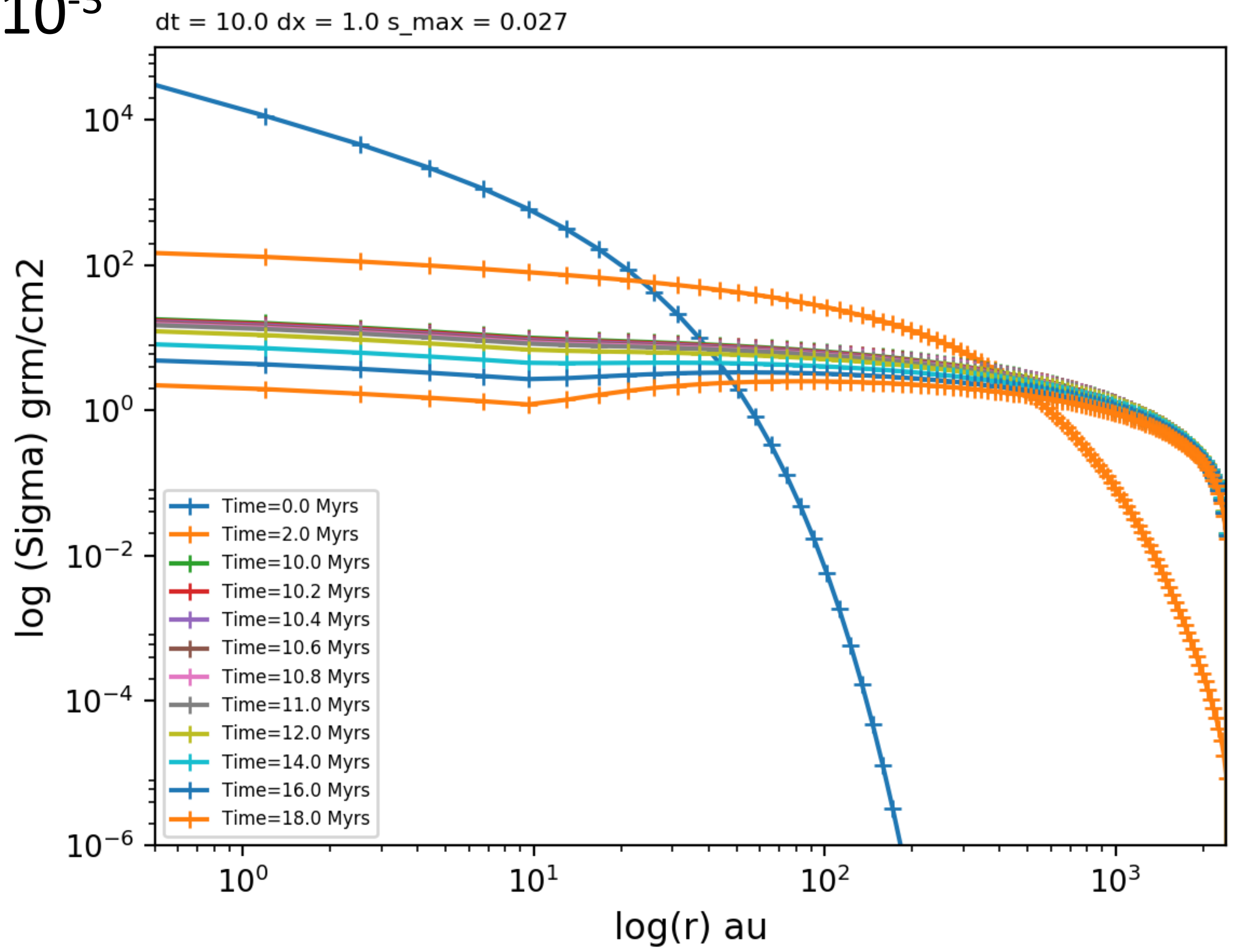
- Discretize the problem.
- Get derivatives using finite differences.
- Iterate solution selecting one numerical scheme.




- ODE: we need to know initial condition.
- PDE: we need in addition behavior of **solution at the boundaries**.

# Boundary Conditions

- $r_{min}$=2.5 $10^{-3}$

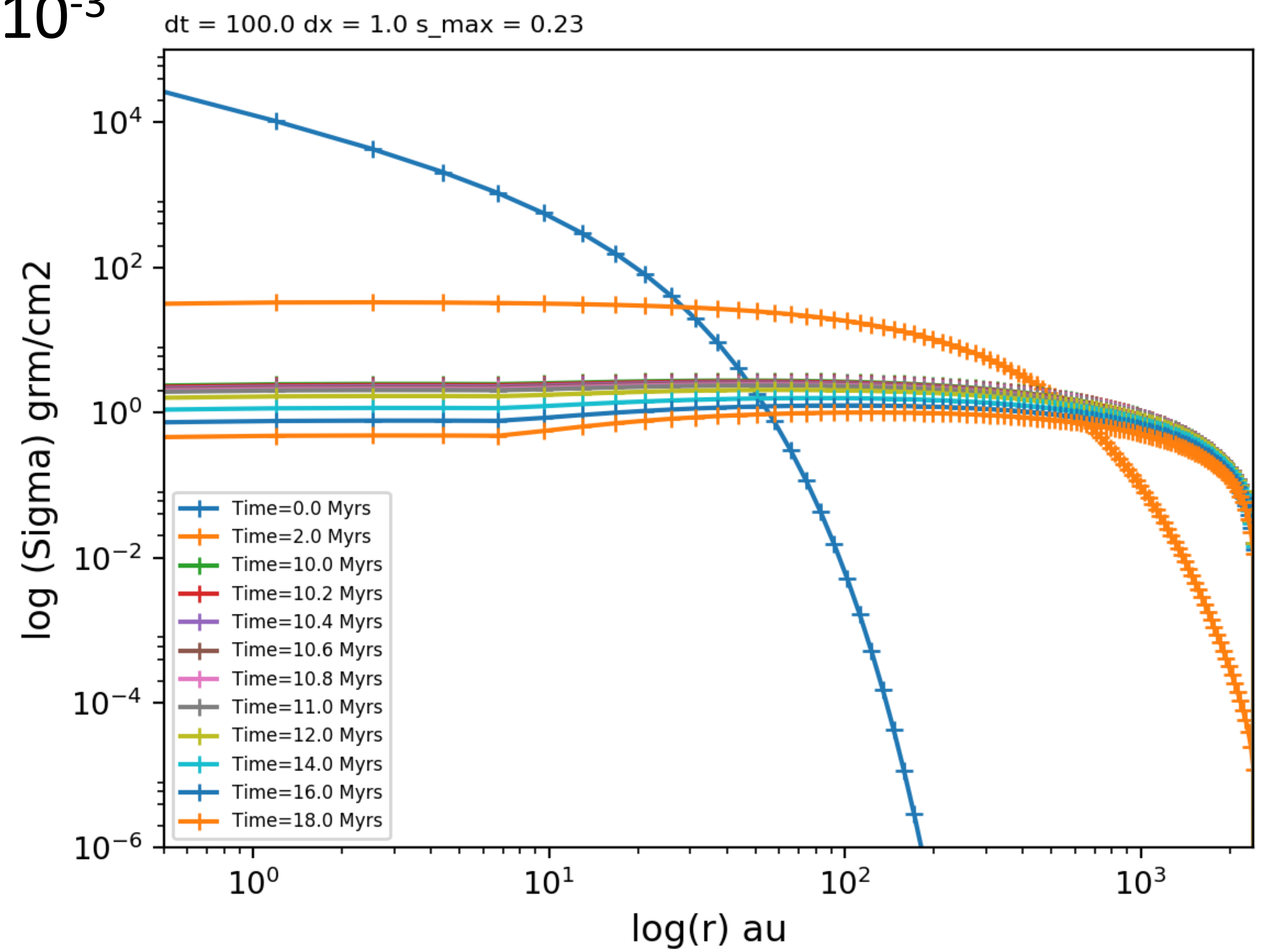


# Discretization

- To characterize what we are doing, we need to address (see also Lax equivalence theorem):
  - **Consistency**: the finite difference representation converges to the PDE we are trying to solve and the space and time steps tend to zero.
  - **Stability**: the difference between the numerical solution and the exact solution remains bounded as we increase the number of steps.
  - **Convergence**: the difference between numerical solutions at fixed point tend to zero as the discretization tend to zero.
- Detail in space steps **synchronized** with detail in time steps.

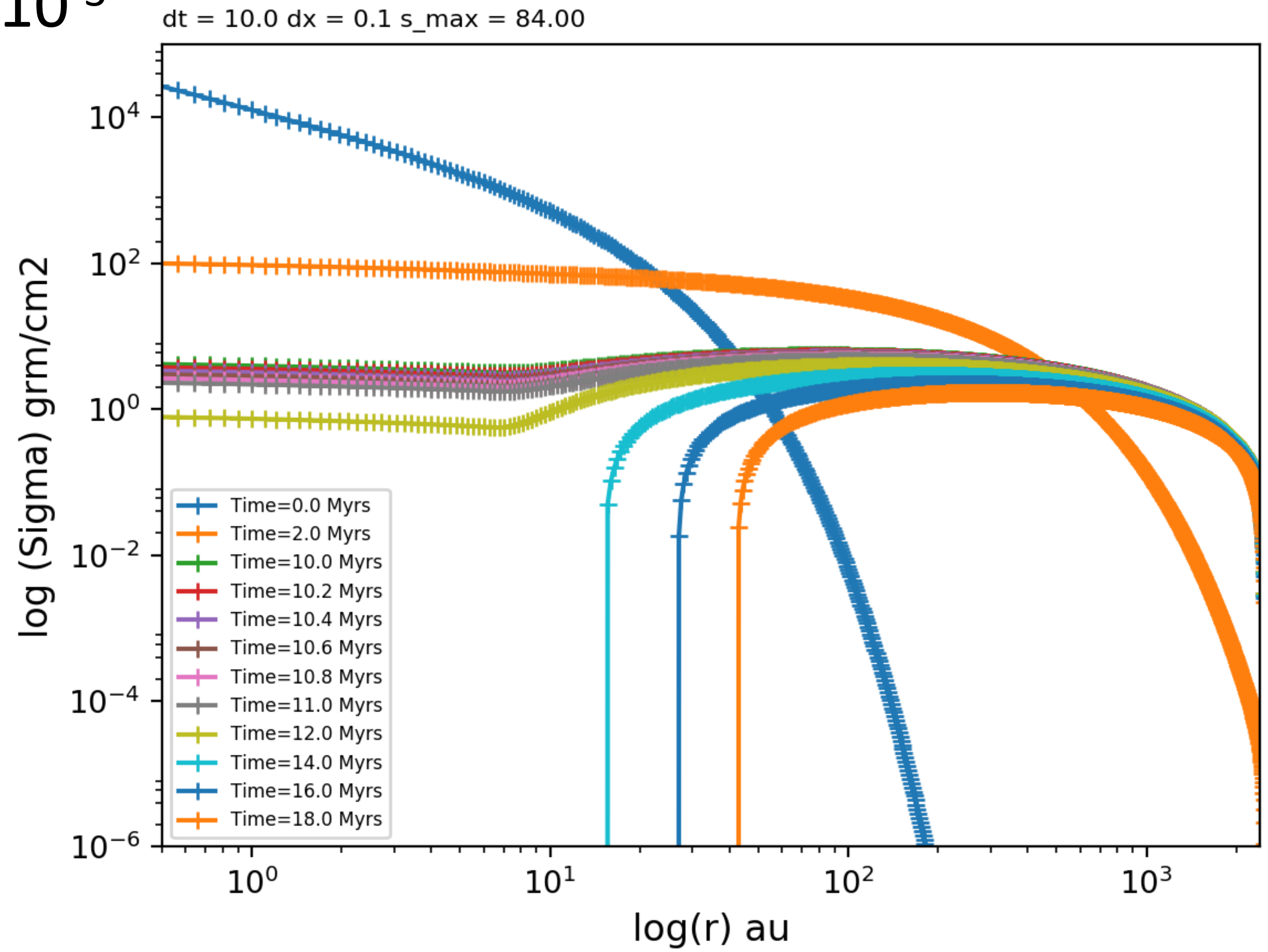
# FTCS and grids (time)

- $r_{min}$=9.0 $10^{-3}$



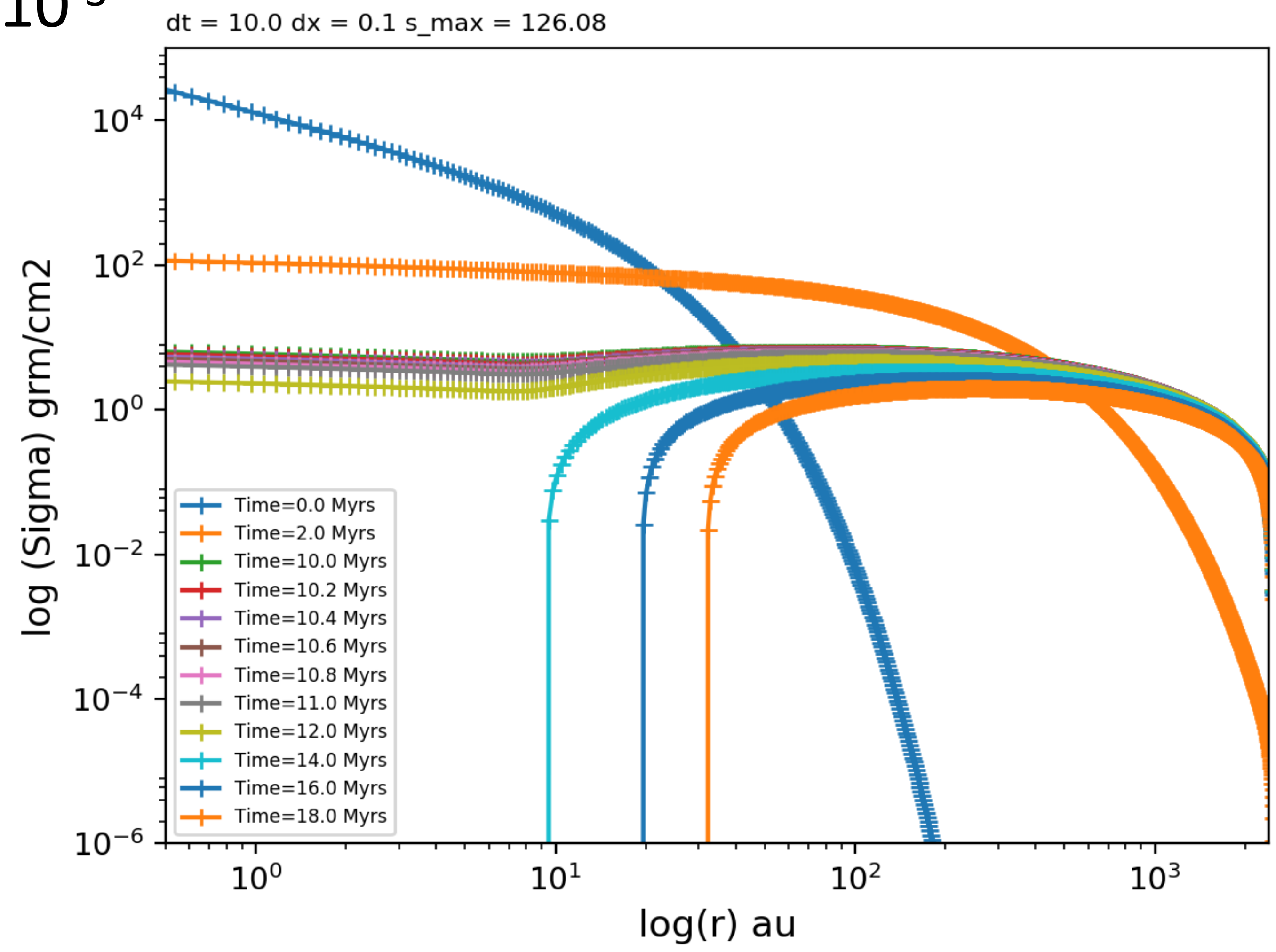
# FTCS and grids (spatial)

- $r_{min}$=2.5 $10^{-3}$



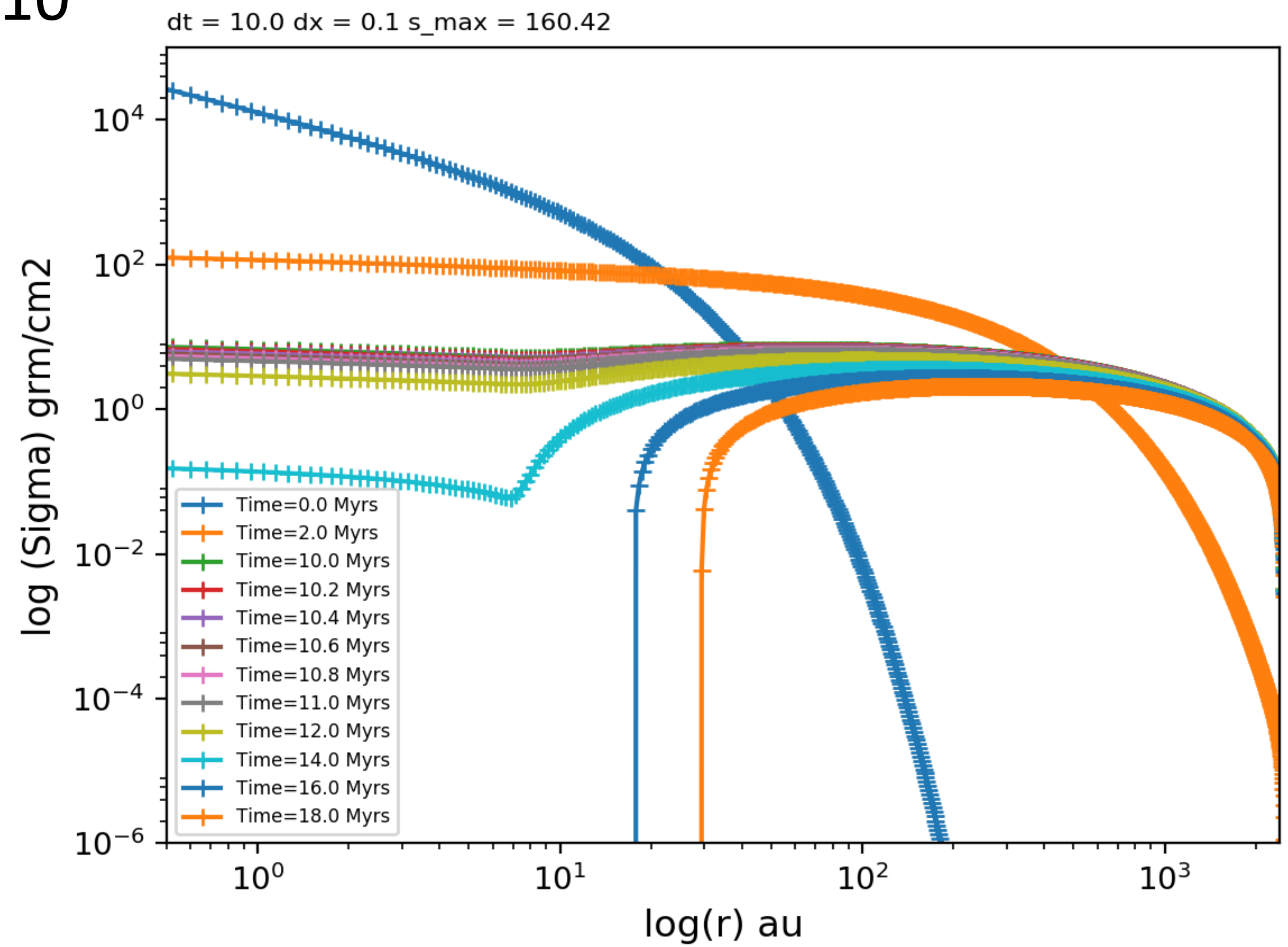
# FTCS and uniform grids (spatial)

- $r_{min}$=1.0 $10^{-3}$



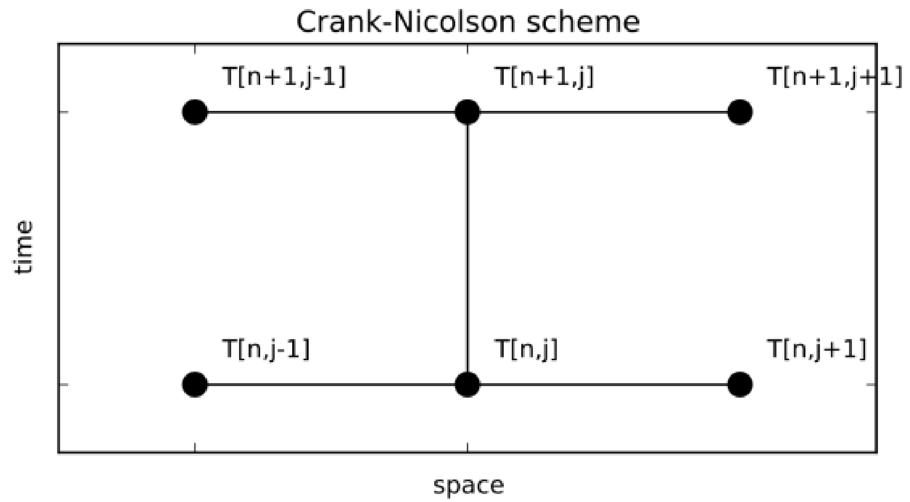
# FTCS and uniform grids (spatial)

- $r_{min}$=0.5 $10^{-3}$



# More sophisticated integrator

- New methods may be used,
  - Crank-Nicolson?
- Logarithmic distances and boundary conditions,
  - Adaptive Finite Elements?




- Worthy the effort?

# More sophisticated model

- We have assumed a phenomenological approach: the disc is turbulent and that this results in an **effective (turbulent) viscosity**.
- Shakura & Sunyeav (1973) $\alpha$ prescription describes the “effective viscosity”.

- **Simple UV-switch model problems**: large ionizing flux needed, outer discs dispersed too slowly (compared with observations).
- When inner disc is drained it may be thin to ionizing radiation. Further refinements needed.

# Context (Reprise)

- If the system to be solved is **only known approximately**, it is meaningless to try to solve it with great accuracy.
- It can be enough **to solve the approximate system** and to focus on assuring that the numerical resolution will not introduce large errors, keeping them, at least, of the same order of magnitude than the error introduced by having an approximated model.
- We should know at least what are the **implication**s of their existence for the intended forecast.

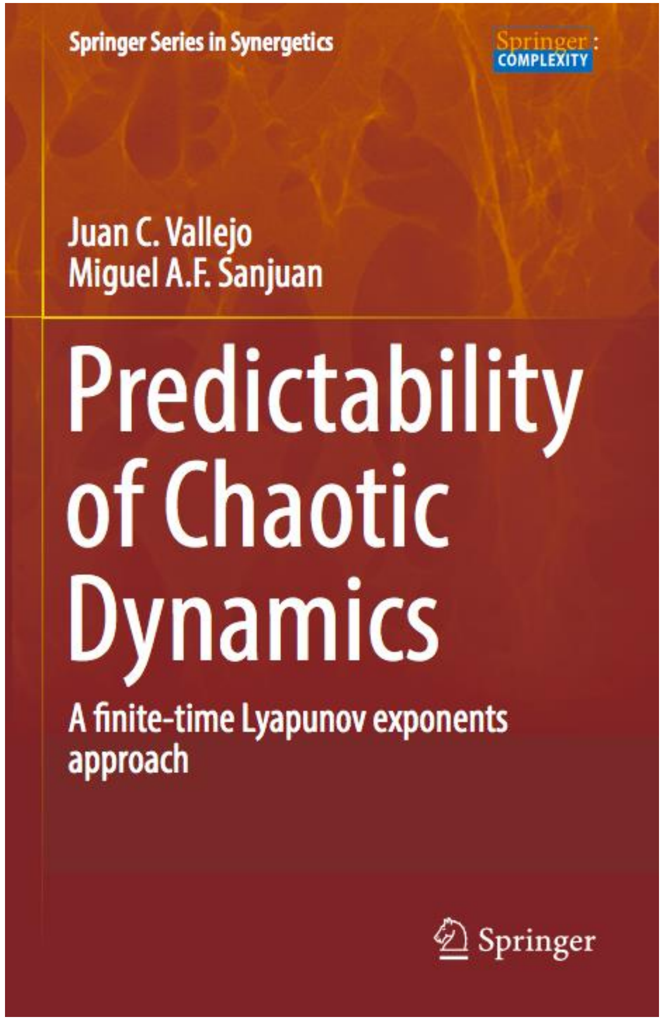
- This talk present**ed** two different approaches within this context.

# Underlying issues...

- Are expensive **schemes** (ODE high precision schemes, PDE Crank-Nicolson, others…) worthy to implement in our model?
- The **model** assumptions are introducing larger errors?

- Overall behavior and predictions of our **combined** model+integrator are ok?
- Is it a too simplified numerical method or is it a too simplified physical model?

# More info...?



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# Thank you for your attention.

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