

Implementation of DME-3rnds-8vars-32bits-sign

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The **DME-3rnds-8vars-32bits-sign** is a signature scheme based on the composition of three different types of polynomial maps $\mathbb{F}_{2^{32}}^8 \rightarrow \mathbb{F}_{2^{32}}^8$ that are bijective almost everywhere: linear maps, affine shifts, and exponential maps. The individual maps form the secret key, and the composition of the maps, which is given by eight polynomials in $\mathbb{F}_{2^{32}}[x_1, \dots, x_8]$ is the public key. The signature is obtained by mapping the message to $\mathbb{F}_{2^{32}}^8$ using a hash function (and a PSS padding with 64 random bits) and then applying the decryption map to get a signature of 256 bits (32 bytes).

1 Mathematical description of DME-3rnds-8vars-32bits-sign

Let $q = 2^{32}$ and let \mathbb{F}_q be a finite field with q elements. Consider an irreducible monic polynomial $p(u) = u^2 + p_1u + p_0 \in \mathbb{F}_q[u]$. The quotient ring $\mathbb{F}_q[u]/\langle p(u) \rangle$ defines a field of q^2 elements, which we denote \mathbb{F}_{q^2} . The map $\phi : \mathbb{F}_q^2 \rightarrow \mathbb{F}_{q^2}$ given by

$$\begin{bmatrix} x \\ y \end{bmatrix} \mapsto x + yu$$

is a bijection. This map can be extended naturally to a map $\bar{\phi} : \mathbb{F}_q^8 \rightarrow (\mathbb{F}_{q^2})^4$

$$\bar{\phi} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} \phi \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ \phi \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \\ \phi \begin{bmatrix} x_5 \\ x_6 \end{bmatrix} \\ \phi \begin{bmatrix} x_7 \\ x_8 \end{bmatrix} \end{bmatrix}$$

which is also a bijection.

For any matrix $M \in \mathbb{Z}^{4 \times 4}$, we define the exponential map $E_M : (\mathbb{F}_{q^2}^*)^4 \rightarrow (\mathbb{F}_{q^2}^*)^4$ given by

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \mapsto \begin{bmatrix} x_1^{m_{11}} x_2^{m_{12}} x_3^{m_{13}} x_4^{m_{14}} \\ x_1^{m_{21}} x_2^{m_{22}} x_3^{m_{23}} x_4^{m_{24}} \\ x_1^{m_{31}} x_2^{m_{32}} x_3^{m_{33}} x_4^{m_{34}} \\ x_1^{m_{41}} x_2^{m_{42}} x_3^{m_{43}} x_4^{m_{44}} \end{bmatrix}.$$

The following result summarizes the properties of the exponential maps that are needed for the **DME-3rnds-8vars-32bits-sign** cryptosystem.

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Lemma 1.1. *Let $M_1, M_2 \in \mathbb{Z}^{4 \times 4}$. Then:*

1. $E_{M_1} \circ E_{M_2} = E_{M_1 \cdot M_2}$.
2. $M_1 \equiv M_2 \pmod{q^2 - 1} \Rightarrow E_{M_1} = E_{M_2}$.
3. $M_1 \cdot M_2 \equiv \text{Id} \pmod{q^2 - 1} \Rightarrow E_{M_1} \circ E_{M_2} = \text{Id}$.
4. $\gcd(\det(M_1), q^2 - 1) = 1 \Rightarrow E_{M_1}$ is invertible.

If no entry of the matrix M is negative, then E_M can be extended to a map $\overline{E_M} : (\mathbb{F}_{q^2})^4 \rightarrow (\mathbb{F}_{q^2})^4$ with the same formula and setting $0^0 = 1$. It should be noted that the extended maps $\overline{E_M}$ fail in general to be bijections, even if $\gcd(\det(M), q^2 - 1) = 1$.

In **DME-3rnds-8vars-32bits-sign**, we have three exponential maps E_1 , E_2 and E_3 , whose matrices are

$$M_1 = \begin{bmatrix} 2^{a_0} & 0 & 0 & 0 \\ 2^{a_1} & 2^{a_2} & 0 & 0 \\ 0 & 0 & 2^{a_3} & 0 \\ 0 & 0 & 2^{a_4} & 2^{a_5} \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 2^{b_0} & 0 & 0 & 2^{b_1} \\ 0 & 2^{b_2} & 0 & 0 \\ 0 & 2^{b_3} & 2^{b_4} & 0 \\ 0 & 0 & 0 & 2^{b_5} \end{bmatrix},$$

$$M_3 = \begin{bmatrix} 2^{c_0} & 2^{c_1} & 0 & 0 \\ 0 & 2^{c_2} & 0 & 2^{c_3} \\ 0 & 2^{c_4} & 0 & 2^{c_5} \\ 0 & 0 & 2^{c_6} & 2^{c_7} \end{bmatrix},$$

respectively, with $a_0, \dots, a_5, b_0, \dots, b_5, c_0, \dots, c_7 \in [0, 63]$ such that

$$\begin{aligned} c_1 &\equiv a_0 + b_0 + c_0 - a_1 - b_2 \pmod{64}, \\ c_7 &\equiv a_3 + b_4 + c_6 - a_4 - b_5 \pmod{64}, \\ c_4 &\equiv c_2 + c_5 - c_3 + 57 \pmod{64}. \end{aligned}$$

It is easy to verify that the three matrices M_1 , M_2 and M_3 satisfy condition 4 of lemma 1.1.

In **DME-3rnds-8vars-32bits-sign**, we also needs four invertible linear maps $L_1, L_2, L_3, L_4 : \mathbb{F}_q^8 \rightarrow \mathbb{F}_q^8$, each of which has a four 2×2 block structure

$$L_i \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} L_{i1} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ L_{i2} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \\ L_{i3} \begin{bmatrix} x_5 \\ x_6 \end{bmatrix} \\ L_{i4} \begin{bmatrix} x_7 \\ x_8 \end{bmatrix} \end{bmatrix}$$

with $L_{ij} \in \mathbb{F}_q^{2 \times 2}$ and $\det(L_{ij}) \neq 0$.

In addition to the linear maps, we have three affine shifts $A_2, A_3, A_4 : \mathbb{F}_q^8 \rightarrow \mathbb{F}_q^8$ given by

$$A_i \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} x_1 + A_{i1} \\ x_2 + A_{i2} \\ x_3 + A_{i3} \\ x_4 + A_{i4} \\ x_5 + A_{i5} \\ x_6 + A_{i6} \\ x_7 + A_{i7} \\ x_8 + A_{i8} \end{bmatrix}$$

with $A_{ij} \in \mathbb{F}_q$.

The secret key consists of the four linear maps L_1, L_2, L_3, L_4 , the three affine shifts A_2, A_3, A_4 and the three exponential maps E_1, E_2, E_3 . The following composition

$$A_4 \circ L_4 \circ \bar{\phi}^{-1} \circ \bar{E}_3 \circ \bar{\phi} \circ A_3 \circ L_3 \circ \bar{\phi}^{-1} \circ \bar{E}_2 \circ \bar{\phi} \circ A_2 \circ L_2 \circ \bar{\phi}^{-1} \circ \bar{E}_1 \circ \bar{\phi} \circ L_1$$

defines a map **dme-enc** : $\mathbb{F}_q^8 \rightarrow \mathbb{F}_q^8$.

Let $D \subseteq \mathbb{F}_q^8$ be the set of $x \in \mathbb{F}_q^8$ such that

$$\begin{aligned} &(\bar{\phi}^{-1} \circ L_1)(x), \\ &(\bar{\phi}^{-1} \circ A_2 \circ L_2 \circ \bar{\phi}^{-1} \circ \bar{E}_1 \circ \bar{\phi} \circ L_1)(x), \\ &(\bar{\phi}^{-1} \circ A_3 \circ L_3 \circ \bar{\phi}^{-1} \circ \bar{E}_2 \circ \bar{\phi} \circ A_2 \circ L_2 \circ \bar{\phi}^{-1} \circ \bar{E}_1 \circ \bar{\phi} \circ L_1)(x) \end{aligned}$$

belong to $(\mathbb{F}_{q^2}^*)^4$, i.e. do not have a zero entry. Let $E = \mathbf{dme-enc}(D) \subseteq \mathbb{F}_q^8$. By construction, the restriction **dme-enc** : $D \rightarrow E$ is a bijection.

Lemma 1.2. $|D| \geq 3(q^2 - 1)^4 - 2q^8 \geq q^8 - 12q^6$. In particular, the probability that a randomly chosen $x \in \mathbb{F}_q^8$ (with a uniform distribution) does not belong to D is at most $12q^{-2} < 2^{-60}$.

The main property of the map **dme-enc** is that it can be given by polynomials (this fact can be proven by following the sequence of maps that define **dme-enc**, starting with 8 variables x_1, \dots, x_8). More precisely, there exists $p_1, \dots, p_8 \in \mathbb{F}_q[x_1, \dots, x_8]$ such that

$$\mathbf{dme-enc} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} p_1(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) \\ p_2(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) \\ p_3(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) \\ p_4(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) \\ p_5(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) \\ p_6(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) \\ p_7(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) \\ p_8(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) \end{bmatrix}$$

where p_1, p_2, p_7, p_8 having 65 monomials each and p_3, p_4, p_5, p_6 having 25 monomials each.

Define the integers $f_0, \dots, f_{15} \in [0, 31]$ as

$$\begin{aligned} f_0 &= a_0 + b_0 + c_0 \bmod 32 \\ f_1 &= a_1 + b_2 + c_2 \bmod 32 \\ f_2 &= a_1 + b_2 + c_4 \bmod 32 \\ f_3 &= a_1 + b_2 + c_6 \bmod 32 \\ f_4 &= a_2 + a_0 + b_0 - a_1 + c_0 \bmod 32 \\ f_5 &= a_2 + b_2 + c_2 \bmod 32 \\ f_6 &= a_2 + b_2 + c_4 \bmod 32 \\ f_7 &= a_2 + b_2 + c_6 \bmod 32 \\ f_8 &= a_4 + b_5 + c_1 \bmod 32 \\ f_9 &= a_4 + b_5 + c_3 \bmod 32 \\ f_{10} &= a_4 + b_5 + c_5 \bmod 32 \\ f_{11} &= a_3 + b_3 + c_7 \bmod 32 \\ f_{12} &= a_5 + b_5 + c_1 \bmod 32 \\ f_{13} &= a_5 + b_5 + c_3 \bmod 32 \\ f_{14} &= a_5 + b_5 + c_5 \bmod 32 \\ f_{15} &= a_5 + a_3 + b_3 - a_4 + c_7 \bmod 32 \end{aligned}$$

and consider the expressions

$$\begin{array}{llll}
z_0 = x_1^{2f_0} & z_1 = x_1^{2f_1} & z_2 = x_1^{2f_2} & z_3 = x_1^{2f_3} \\
z_4 = x_2^{2f_0} & z_5 = x_2^{2f_1} & z_6 = x_2^{2f_2} & z_7 = x_1^{2f_3} \\
z_8 = x_3^{2f_4} & z_9 = x_3^{2f_5} & z_{10} = x_3^{2f_6} & z_{11} = x_3^{2f_7} \\
z_{12} = x_4^{2f_4} & z_{13} = x_4^{2f_5} & z_{14} = x_4^{2f_6} & z_{15} = x_4^{2f_7} \\
z_{16} = x_5^{2f_8} & z_{17} = x_5^{2f_9} & z_{18} = x_5^{2f_{10}} & z_{19} = x_5^{2f_{11}} \\
z_{20} = x_6^{2f_8} & z_{21} = x_6^{2f_9} & z_{22} = x_6^{2f_{10}} & z_{23} = x_6^{2f_{11}} \\
z_{24} = x_7^{2f_{12}} & z_{25} = x_7^{2f_{13}} & z_{26} = x_7^{2f_{14}} & z_{27} = x_7^{2f_{15}} \\
z_{28} = x_8^{2f_{12}} & z_{29} = x_8^{2f_{13}} & z_{30} = x_8^{2f_{14}} & z_{31} = x_8^{2f_{15}}
\end{array}$$

A careful study of p_1 and p_2 show that the 65 monomials are exactly

$$\begin{array}{lll}
m_{1,1} = z_{24}z_{16}z_8z_0^2 & m_{1,2} = z_{28}z_{16}z_8z_0^2 & m_{1,3} = z_{24}z_{20}z_8z_0^2 \\
m_{1,4} = z_{28}z_{20}z_8z_0^2 & m_{1,5} = z_8z_0^2 & m_{1,6} = z_{24}z_{16}z_{12}z_0^2 \\
m_{1,7} = z_{28}z_{16}z_{12}z_0^2 & m_{1,8} = z_{24}z_{20}z_{12}z_0^2 & m_{1,9} = z_{28}z_{20}z_{12}z_0^2 \\
m_{1,10} = z_{12}z_0^2 & m_{1,11} = z_{24}z_{16}z_8z_4z_0 & m_{1,12} = z_{28}z_{16}z_8z_4z_0 \\
m_{1,13} = z_{24}z_{20}z_8z_4z_0 & m_{1,14} = z_{28}z_{20}z_8z_4z_0 & m_{1,15} = z_8z_4z_0 \\
m_{1,16} = z_{24}z_{16}z_{12}z_4z_0 & m_{1,17} = z_{28}z_{16}z_{12}z_4z_0 & m_{1,18} = z_{24}z_{20}z_{12}z_4z_0 \\
m_{1,19} = z_{28}z_{20}z_{12}z_4z_0 & m_{1,20} = z_{12}z_4z_0 & m_{1,21} = z_{24}z_{16}z_0 \\
m_{1,22} = z_{28}z_{16}z_0 & m_{1,23} = z_{24}z_{20}z_0 & m_{1,24} = z_{28}z_{20}z_0 \\
m_{1,25} = z_0 & m_{1,26} = z_{24}z_{16}z_8z_4^2 & m_{1,27} = z_{28}z_{16}z_8z_4^2 \\
m_{1,28} = z_{24}z_{20}z_8z_4^2 & m_{1,29} = z_{28}z_{20}z_8z_4^2 & m_{1,30} = z_8z_4^2 \\
m_{1,31} = z_{24}z_{16}z_{12}z_4^2 & m_{1,32} = z_{28}z_{16}z_{12}z_4^2 & m_{1,33} = z_{24}z_{20}z_{12}z_4^2 \\
m_{1,34} = z_{28}z_{20}z_{12}z_4^2 & m_{1,35} = z_{12}z_4^2 & m_{1,36} = z_{24}z_{16}z_4 \\
m_{1,37} = z_{28}z_{16}z_4 & m_{1,38} = z_{24}z_{20}z_4 & m_{1,39} = z_{28}z_{20}z_4 \\
m_{1,40} = z_4 & m_{1,41} = z_{24}z_{16}z_8z_0 & m_{1,42} = z_{28}z_{16}z_8z_0 \\
m_{1,43} = z_{24}z_{20}z_8z_0 & m_{1,44} = z_{28}z_{20}z_8z_0 & m_{1,45} = z_8z_0 \\
m_{1,46} = z_{24}z_{16}z_{12}z_0 & m_{1,47} = z_{28}z_{16}z_{12}z_0 & m_{1,48} = z_{24}z_{20}z_{12}z_0 \\
m_{1,49} = z_{28}z_{20}z_{12}z_0 & m_{1,50} = z_{12}z_0 & m_{1,51} = z_{24}z_{16}z_8z_4 \\
m_{1,52} = z_{28}z_{16}z_8z_4 & m_{1,53} = z_{24}z_{20}z_8z_4 & m_{1,54} = z_{28}z_{20}z_8z_4 \\
m_{1,55} = z_8z_4 & m_{1,56} = z_{24}z_{16}z_{12}z_4 & m_{1,57} = z_{28}z_{16}z_{12}z_4 \\
m_{1,58} = z_{24}z_{20}z_{12}z_4 & m_{1,59} = z_{28}z_{20}z_{12}z_4 & m_{1,60} = z_{12}z_4 \\
m_{1,61} = z_{24}z_{16} & m_{1,62} = z_{28}z_{16} & m_{1,63} = z_{24}z_{20} \\
m_{1,64} = z_{28}z_{20} & m_{1,65} = 1 &
\end{array}$$

Similarly, the 25 monomials that appear in p_3 and p_4 are

$$\begin{array}{lll}
m_{2,1} = z_{25}z_{17}z_9z_1 & m_{2,2} = z_{29}z_{17}z_9z_1 & m_{2,3} = z_{25}z_{21}z_9z_1 \\
m_{2,4} = z_{29}z_{21}z_9z_1 & m_{2,5} = z_9z_1 & m_{2,6} = z_{25}z_{17}z_{13}z_1 \\
m_{2,7} = z_{29}z_{17}z_{13}z_1 & m_{2,8} = z_{25}z_{21}z_{13}z_1 & m_{2,9} = z_{29}z_{21}z_{13}z_1 \\
m_{2,10} = z_{13}z_1 & m_{2,11} = z_{25}z_{17}z_9z_5 & m_{2,12} = z_{29}z_{17}z_9z_5 \\
m_{2,13} = z_{25}z_{21}z_9z_5 & m_{2,14} = z_{29}z_{21}z_9z_5 & m_{2,15} = z_9z_5 \\
m_{2,16} = z_{25}z_{17}z_{13}z_5 & m_{2,17} = z_{29}z_{17}z_{13}z_5 & m_{2,18} = z_{25}z_{21}z_{13}z_5 \\
m_{2,19} = z_{29}z_{21}z_{13}z_5 & m_{2,20} = z_{13}z_5 & m_{2,21} = z_{25}z_{17} \\
m_{2,22} = z_{29}z_{17} & m_{2,23} = z_{25}z_{21} & m_{2,24} = z_{29}z_{21} \\
m_{2,25} = 1 & &
\end{array}$$

the 25 monomials that appear in p_5 and p_6 are

$$\begin{aligned}
m_{3,1} &= z_{26}z_{18}z_{10}z_2 & m_{3,2} &= z_{30}z_{18}z_{10}z_2 & m_{3,3} &= z_{26}z_{22}z_{10}z_2 \\
m_{3,4} &= z_{30}z_{22}z_{10}z_2 & m_{3,5} &= z_{10}z_2 & m_{3,6} &= z_{26}z_{18}z_{14}z_2 \\
m_{3,7} &= z_{30}z_{18}z_{14}z_2 & m_{3,8} &= z_{26}z_{22}z_{14}z_2 & m_{3,9} &= z_{30}z_{22}z_{14}z_2 \\
m_{3,10} &= z_{14}z_2 & m_{3,11} &= z_{26}z_{18}z_{10}z_6 & m_{3,12} &= z_{30}z_{18}z_{10}z_6 \\
m_{3,13} &= z_{26}z_{22}z_{10}z_6 & m_{3,14} &= z_{30}z_{22}z_{10}z_6 & m_{3,15} &= z_{10}z_6 \\
m_{3,16} &= z_{26}z_{18}z_{14}z_6 & m_{3,17} &= z_{30}z_{18}z_{14}z_6 & m_{3,18} &= z_{26}z_{22}z_{14}z_6 \\
m_{3,19} &= z_{30}z_{22}z_{14}z_6 & m_{3,20} &= z_{14}z_6 & m_{3,21} &= z_{26}z_{18} \\
m_{3,22} &= z_{30}z_{18} & m_{3,23} &= z_{26}z_{22} & m_{3,24} &= z_{30}z_{22} \\
m_{3,25} &= 1
\end{aligned}$$

and the 65 monomials that appear in p_7 and p_8 are

$$\begin{aligned}
m_{4,1} &= z_{27}z_{19}z_{11}z_3 & m_{4,2} &= z_{31}z_{19}z_{11}z_3 & m_{4,3} &= z_{27}z_{23}z_{19}z_{11}z_3 \\
m_{4,4} &= z_{31}z_{23}z_{19}z_{11}z_3 & m_{4,5} &= z_{19}z_{11}z_3 & m_{4,6} &= z_{27}z_{23}z_{11}z_3 \\
m_{4,7} &= z_{31}z_{23}z_{11}z_3 & m_{4,8} &= z_{23}z_{11}z_3 & m_{4,9} &= z_{27}z_{19}z_{11}z_3 \\
m_{4,10} &= z_{31}z_{19}z_{11}z_3 & m_{4,11} &= z_{27}z_{23}z_{11}z_3 & m_{4,12} &= z_{31}z_{23}z_{11}z_3 \\
m_{4,13} &= z_{11}z_3 & m_{4,14} &= z_{27}z_{19}z_{15}z_3 & m_{4,15} &= z_{31}z_{19}z_{15}z_3 \\
m_{4,16} &= z_{27}z_{23}z_{19}z_{15}z_3 & m_{4,17} &= z_{31}z_{23}z_{19}z_{15}z_3 & m_{4,18} &= z_{19}z_{15}z_3 \\
m_{4,19} &= z_{27}z_{23}z_{15}z_3 & m_{4,20} &= z_{31}z_{23}z_{15}z_3 & m_{4,21} &= z_{23}z_{15}z_3 \\
m_{4,22} &= z_{27}z_{19}z_{15}z_3 & m_{4,23} &= z_{31}z_{19}z_{15}z_3 & m_{4,24} &= z_{27}z_{23}z_{15}z_3 \\
m_{4,25} &= z_{31}z_{23}z_{15}z_3 & m_{4,26} &= z_{15}z_3 & m_{4,27} &= z_{27}z_{19}z_{11}z_7 \\
m_{4,28} &= z_{31}z_{19}z_{11}z_7 & m_{4,29} &= z_{27}z_{23}z_{19}z_{11}z_7 & m_{4,30} &= z_{31}z_{23}z_{19}z_{11}z_7 \\
m_{4,31} &= z_{19}z_{11}z_7 & m_{4,32} &= z_{27}z_{23}z_{11}z_7 & m_{4,33} &= z_{31}z_{23}z_{11}z_7 \\
m_{4,34} &= z_{23}z_{11}z_7 & m_{4,35} &= z_{27}z_{19}z_{11}z_7 & m_{4,36} &= z_{31}z_{19}z_{11}z_7 \\
m_{4,37} &= z_{27}z_{23}z_{11}z_7 & m_{4,38} &= z_{31}z_{23}z_{11}z_7 & m_{4,39} &= z_{11}z_7 \\
m_{4,40} &= z_{27}z_{19}z_{15}z_7 & m_{4,41} &= z_{31}z_{19}z_{15}z_7 & m_{4,42} &= z_{27}z_{23}z_{19}z_{15}z_7 \\
m_{4,43} &= z_{31}z_{23}z_{19}z_{15}z_7 & m_{4,44} &= z_{19}z_{15}z_7 & m_{4,45} &= z_{27}z_{23}z_{15}z_7 \\
m_{4,46} &= z_{31}z_{23}z_{15}z_7 & m_{4,47} &= z_{23}z_{15}z_7 & m_{4,48} &= z_{27}z_{19}z_{15}z_7 \\
m_{4,49} &= z_{31}z_{19}z_{15}z_7 & m_{4,50} &= z_{27}z_{23}z_{15}z_7 & m_{4,51} &= z_{31}z_{23}z_{15}z_7 \\
m_{4,52} &= z_{15}z_7 & m_{4,53} &= z_{27}z_{19}z_2 & m_{4,54} &= z_{31}z_{19}z_2 \\
m_{4,55} &= z_{27}z_{23}z_{19} & m_{4,56} &= z_{31}z_{23}z_{19} & m_{4,57} &= z_{19} \\
m_{4,58} &= z_{27}z_{23}z_2 & m_{4,59} &= z_{31}z_{23}z_2 & m_{4,60} &= z_{23} \\
m_{4,61} &= z_{27}z_{19} & m_{4,62} &= z_{31}z_{19} & m_{4,63} &= z_{27}z_{23} \\
m_{4,64} &= z_{31}z_{23} & m_{4,65} &= 1
\end{aligned}$$

Using the notation above, the polynomials p_1, \dots, p_8 can be written as

$$\begin{aligned}
p_1 &= \sum_{i=1}^{65} p_{1,i} m_{1,i} & p_2 &= \sum_{i=1}^{65} p_{2,i} m_{1,i} \\
p_3 &= \sum_{i=1}^{25} p_{3,i} m_{2,i} & p_4 &= \sum_{i=1}^{25} p_{4,i} m_{2,i} \\
p_5 &= \sum_{i=1}^{25} p_{5,i} m_{3,i} & p_6 &= \sum_{i=1}^{25} p_{6,i} m_{3,i} \\
p_7 &= \sum_{i=1}^{65} p_{7,i} m_{4,i} & p_8 &= \sum_{i=1}^{65} p_{8,i} m_{4,i}
\end{aligned}$$

and the public key is just these eight polynomials (which are encoded by the list of 360 coefficients and the values f_0, \dots, f_{15}).

Let M_1^{-1} , M_2^{-1} , and M_3^{-1} be the inverses of M_1 , M_2 , and M_3 modulo $q^2 - 1$, respectively, with their entries reduced to the interval $[0, q^2 - 1)$. Let $E_1^{-1}, E_2^{-1}, E_3^{-1} : (\mathbb{F}_{q^2}^*)^4 \rightarrow (\mathbb{F}_{q^2}^*)^4$ the corresponding exponential maps and $\overline{E_1^{-1}}, \overline{E_2^{-1}}, \overline{E_3^{-1}} : (\mathbb{F}_{q^2})^4 \rightarrow (\mathbb{F}_{q^2})^4$ their extensions. The following composition

$$L_1^{-1} \circ \overline{\phi}^{-1} \circ \overline{E_1^{-1}} \circ \overline{\phi} \circ L_2^{-1} \circ A_2^{-1} \circ \overline{\phi}^{-1} \circ \overline{E_2^{-1}} \circ \overline{\phi} \circ L_3^{-1} \circ A_3^{-1} \circ \overline{\phi}^{-1} \circ \overline{E_3^{-1}} \circ \overline{\phi} \circ L_4^{-1} \circ A_4^{-1}$$

defines a map **dme-dec** : $\mathbb{F}_q^8 \rightarrow \mathbb{F}_q^8$. By construction, we have that **dme-dec** maps E to D and, restricted to those sets, is the inverse of **dme-enc**. It is easy to verify that E is exactly the set of $y \in \mathbb{F}_q^8$ such

that

$$\begin{aligned} &(\bar{\phi} \circ L_4^{-1} \circ A_4^{-1})(y), \\ &(\bar{\phi} \circ L_3^{-1} \circ A_3^{-1} \circ \bar{\phi}^{-1} \circ \overline{E_3^{-1}} \circ \bar{\phi} \circ L_4^{-1} \circ A_4^{-1})(y), \\ &(\bar{\phi} \circ L_2^{-1} \circ A_2^{-1} \circ \bar{\phi}^{-1} \circ \overline{E_2^{-1}} \circ \bar{\phi} \circ L_3^{-1} \circ A_3^{-1} \circ \bar{\phi}^{-1} \circ \overline{E_3^{-1}} \circ \bar{\phi} \circ L_4^{-1} \circ A_4^{-1})(y) \end{aligned}$$

belong to $(\mathbb{F}_{q^2}^*)^4$, i.e. do not have a zero entry.

The cryptographic assumption in **DME-3rnds-8vars-32bits-sign** is that, for any $y \in E$, the system of eight polynomial equations in eight unknowns

$$\begin{aligned} p_1(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) &= y_1 \\ p_2(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) &= y_2 \\ p_3(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) &= y_3 \\ p_4(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) &= y_4 \\ p_5(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) &= y_5 \\ p_6(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) &= y_6 \\ p_7(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) &= y_7 \\ p_8(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) &= y_8 \end{aligned}$$

is hard to solve. In particular, this implies that it is not feasible to compute a secret key corresponding to a given public key.

The **dme-sign** : $\{0, 1\}^* \rightarrow \{0, 1\}^* \times \{0, 1\}^{256}$ map of the **DME-3rnds-8vars-32bits-sign** scheme, as required by the API, returns (m, s) where m is the original message and the signature s is obtained by first applying a PSS-SHA3 padding (with 64 random bits), then reading the 256 bit sequence as a vector in \mathbb{F}_q^8 , applying **dme-dec**, and lastly, interpreting the resulting vector as a 256 bit sequence. The **dme-open** : $\{0, 1\}^* \times \{0, 1\}^{256} \rightarrow \{0, 1\}^* \cup \{error\}$ reverses the procedure above using **dme-enc** and checks that the signature is legitimate. The details of these algorithms are given in the next section.

2 Implementation details of DME-3rnds-8vars-32bits-sign

The field of $q = 2^{32}$ is implemented as the quotient ring

$$\mathbb{F}_q = \mathbb{F}_2[t] / \langle t^{32} + t^{11} + t^4 + t + 1 \rangle,$$

and the monic irreducible polynomial $p(u) \in \mathbb{F}_q[u]$ that defines \mathbb{F}_{q^2} is $p(u) = u^2 + tu + 1$, so we have

$$\mathbb{F}_{q^2} = \mathbb{F}_q[u] / \langle u^2 + tu + 1 \rangle.$$

An element $\alpha = \alpha_{31}t^{31} + \dots + \alpha_1t + \alpha_0 \in \mathbb{F}_q$ can be interpreted as the 32 bits unsigned integer $\text{int}(\alpha) = \alpha_{31}2^{31} + \dots + \alpha_12 + \alpha_0 \in [0, 2^{64} - 1]$. In C99, these fit perfectly in the **uint32_t** type of the standard library. When serialized into bytes, the little-endian convention is used for all integer types. In particular, the element α above, correspond with the sequence of 4 bytes

$$\left(\left\lfloor \frac{\text{int}(\alpha)}{2^{8i}} \right\rfloor \bmod 2^8 \right)$$

for $i = 0, 1, \dots, 3$ in exactly this order. An element $\beta = \beta_0 + \beta_1u \in \mathbb{F}_{q^2}$ is serialized as the 8 byte sequence obtained by serializing first β_0 and then β_1 . Similarly, a matrix $\gamma \in \mathbb{F}_q^{2 \times 2}$ is serialized as the 16 bytes sequence obtained by serializing $\gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22}$ in that order.

The private key is $369 = 16 \cdot 16 + 24 \cdot 4 + 6 + 6 + 5$ bytes long, which correspond to the serialization of the 16 matrices $L_{11}^{-1}, L_{12}^{-1}, \dots, L_{44}^{-1}$, then the serialization of the 24 affine shifts $A_{21}, A_{22}, A_{31}, A_{32}, A_{41}, A_{42}, A_{23}, A_{24}, \dots, A_{47}, A_{48} \in \mathbb{F}_q$, followed by a single byte for each a_0, \dots, a_5 ,

$b_0, \dots, b_5, c_0, c_2, c_3, c_5, c_6$. The coefficients c_1, c_4 and c_7 are not serialized since they can be recovered from the other values.

The public key is $1449 = 360 \cdot 8 + 9$ bytes long, which correspond to the serialization of the coefficients of p_1, p_2, \dots, p_8 followed by a single byte for each $f_0, f_1, f_3, f_5, f_8, f_9, f_{10}, f_{11}, f_{12}$. The values of $f_2, f_4, f_6, f_7, f_{13}, f_{14}, f_{15}$ are not serialized since they can be computed from the other values by

$$\begin{aligned} f_2 &= (f_1 + f_{10} - f_9 + 57) \bmod 32 \\ f_4 &= (f_0 + f_5 - f_1) \bmod 32 \\ f_6 &= (f_5 + f_2 - f_1) \bmod 32 \\ f_7 &= (f_5 + f_3 - f_1) \bmod 32 \\ f_{13} &= (f_{12} + f_9 - f_8) \bmod 32 \\ f_{14} &= (f_{12} + f_{10} - f_8) \bmod 32 \\ f_{15} &= (f_{11} + f_{12} - f_8) \bmod 32 \end{aligned}$$

The **dme-sign** : $\{0, 1\}^* \rightarrow \{0, 1\}^* \times \{0, 1\}^{256}$ map (the secret key is implicit here) is computed by the following procedure:

1. let $msg \in \{0, 1\}^*$ be the input message,
2. choose $r \in \{0, 1\}^{64}$ at random,
3. compute $w = \text{SHA3}(msg||r) \in \{0, 1\}^{128}$,
4. compute $g = \text{SHA3}(w) \oplus (r||0) \in \{0, 1\}^{128}$,
5. compute $s = \text{dme-dec}(w||g) \in \mathbb{F}_q^8 \simeq \{0, 1\}^{256}$,
6. if the call to **dme-dec** detects an invalid input, i.e. a vector that is not in E , go back to step 2,
7. return (msg, s) .

This function is implemented in C99 as **crypto_sign**, with the only difference that the return value is $msg||s$ instead of (msg, s) .

The **dme-open** : $\{0, 1\}^* \times \{0, 1\}^{256} \rightarrow \{0, 1\}^* \cup \{error\}$ map (the public key is implicit here) is computed as follows:

1. let $(msg, s) \in \{0, 1\}^* \times \{0, 1\}^{256}$ be the input message and its corresponding signature,
2. if the interpretation of s as a vector in $(\mathbb{F}_{q^2})^4$ has a zero entry, return *error*,
3. compute $w \in \{0, 1\}^{128}$ and $g \in \{0, 1\}^{128}$ as $w||g = \text{dme-enc}(s)$,
4. compute $r \in \{0, 1\}^{64}$ as the first 64 bits of $\text{SHA3}(w) \oplus g$,
5. if the last 64 bits of $\text{SHA3}(w) \oplus g$ are not all zero, return *error*,
6. if $w \neq \text{SHA3}(msg||r)$, return *error*,
7. otherwise, return the original message msg .

This function is implemented in C99 as **crypto_sign_open**, but the two separate arguments for the message msg and the signature s , the function takes only one with the concatenation of both $msg||s$.

The function **dme-keypair**, which corresponds in the C99 implementation with **crypto_sign_keypair** creates 16 random matrices in $\mathbb{F}_q^{2 \times 2}$, 4 random shifts in \mathbb{F}_q^8 and random values for $a_0, \dots, c_7 \in [0, 127]$ satisfying the restrictions explained in the previous section (for instance, the matrices have to be invertible). With the secret key already chosen, the public key is computed by operating with 8 (symbolic) polynomials until $p_1, \dots, p_8 \in \mathbb{F}_q[x_1, \dots, x_8]$ is obtained. Then both keys are serialized and returned.

3 Timings

On a laptop with a Intel(R) Core(TM) i7-8565U CPU at 1.80GHz, with 8 Gb of RAM, running a Linux Mint 21 x86_64 operating system, the performance of the API primitives (for message of 200 bytes) is given in the following table:

<code>dme-keypair</code>	121 usec
<code>dme-sign</code>	19 usec
<code>dme-open</code>	9 usec

The length of the private key is 369 bytes and the length of the public key is 1449 bytes.