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Corrections to the article:

ON THE BEHAVIOUR OF THE INTERFACE IN NONLINEAR PROCESSES WITH CONVECTION DOMINATING DIFFUSION VIA LAGRANGIAN COORDINATES

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In this note we correct some misprints and mistakes discovered in the paper pointed out in the title. The authors are very grateful to B.H. Gilding who attracted their attention on some obscure points of the paper.

1. **Page 24**. Conditions A.1), A.2) should be made more precise in the following way:

A.1) There exist some positive constants $\lambda\in(0,1)$, and $L>0$ such that $0\le\psi(s)\le Ls^{\lambda}$ for all $s>0$. Moreover

$$\left[s^2\left(\frac{\psi(s)}{s}\right)'\right]'<0 \quad \text{for } s>0 \text{ and } \lim_{s\downarrow 0} s^2\left(\frac{\psi(s)}{s}\right)'=0.$$

A.2) For any $s>0$ we have

$$F(s):=\int_0^s \frac{\sigma\varphi'(\sigma)}{\psi(\sigma)}\,d\sigma<\infty.$$

Moreover, we assume that F is a convex function,

$$0<C\le\frac{s^2\varphi'(s)}{\psi(s)F(s)}\le C', \qquad \left(\frac{F^{\varepsilon}(s)}{s^2\varphi'(s)}\right)'\le 0$$

for some C,C' and $\varepsilon>0$ and $\left(\frac{\left(\frac{\psi(s)}{s}\right)'}{\varphi'(s)}\right)'\ge 0$ for all $s>0$.

2. **Page** 27. As consequence of the new conditions A.1), A.2), conditions B_1), B_2) and B_4) become

B_1) $\quad C_1 \le \frac{a(w)b(w)}{w\psi(F^{-1}(w))} \le C_2 \qquad$ for all $w \ge 0$ and some $C_1, C_2 > 0$,

B_2) $\quad \frac{b'_w}{b} \le 0$ and $\left(\frac{b'_w}{b}\right)'_w \ge 0 \qquad$ for all $w \ge 0$,

B_4) $\quad w\left(\frac{a'_w}{a} + \frac{b'_w}{b}\right) \ge \varepsilon \qquad$ for all $w \ge 0$, where ε is defined in condition A.2).

3. **Page 28**. Assumption (2.9) in Theorem 1 should be replaced by the more restrictive condition

$$-1 \le \frac{dH(u_0(x))}{dx} < \infty \text{ for all } x \in [0,a] \tag{2.9}$$

This correction implies that condition d) at page 27 must be replaced by

$$-1 \le \frac{dw_0}{dp}(p) < \infty \qquad \text{for all } s \in [0,1].$$

A similar modification must be introduced in condition H_4.

4. **Pages 30, 31 and 35**. In the corrected formulation Theorem 1 and Theorem 3 only contain global assumptions on $u_0(x)$ and $w_0(p)$ respectively (and not local assumptions near the points x=a or p=1) in contrast to the published version. Remarks 4, 5 and 8 should be omitted. Those remarks do not have any implication in the rest of the paper.

5. **Page 34**. Proof of Lemma 6. Dropping subindexes we have for $d_n(\equiv d)$:

$$M_n d \equiv \frac{\partial d}{\partial t} - ab\frac{\partial^2 d}{\partial p^2} - \left[\left(2\frac{ab}{w} + 2ab'_w\right)\frac{\partial w}{\partial p} + ab'_w\right]\frac{\partial d}{\partial p} -$$

$$-\frac{ab}{w}\left\{\left(\frac{wb'_w}{b}\right)'_w \frac{\partial w}{\partial p} + \left(\frac{wb'_w}{b}\right)'_w - \frac{b'_w}{b}\right\}\frac{\partial w}{\partial p}\, d -$$

$$- w\left(\frac{a'_w}{a} + \frac{b'_w}{b}\right)d^2 = 0. \tag{4.14}$$

Consider the function

$$D = -\frac{T}{t}\,\frac{C}{(p-\sigma')^2}$$

where C is a positive constant to be chosen. From assumptions B_1), B_3) and Lemma 2 we see that the coefficient of d_p, in (4.14), is bounded. Moreover, in view of B_1), B_2) and Lemma 5 the coefficient of d is nonnegative. Indeed, rewriting it as

$$\nu(\rho,t) \equiv -\frac{ab}{w}\left[\left(\frac{wb'_w}{b}\right)'_w (w_p+1) - \frac{b'_w}{b}\right] w_p$$

we have that if $(w\frac{b'_w}{b})' \ge 0$ then $\nu(p,t)$ is nonnegative due to (4.4), the first inequality in B_2) and Lemma 5. If $(w\frac{b'_w}{b})' < 0$ we use representation

$$\nu(\rho,t) \equiv -w_p\frac{ab}{w}\left[\left(w\frac{b'_w}{b}\right)'_w w_p + w\left(\frac{b'_w}{b}\right)'_w\right]$$

and the nonnegativeness of $\nu(p,t)$ follows from Lemma 5 and B_2). The rest of the proof follows like in the article.

6. **Pages 36-37**. Due to (4.4) formulas (4.20) and (4.21) simplify and take the form

$U_n(p,t) \ge U_n(p,\tau_0)\exp(-L_2(t-\tau_0))$ and $U_n(p,t) \ge F^{-1}\left[F\left(U_n(p,\tau_0)\exp(-L_2(t-\tau_0)\right)\right]$.

Inequality (4.23) becomes

$$\int_0^t W_n(p,\theta)d\theta \ge 0.$$

and it follows from (4.4).

7. We point out some misprints without any influence on the results:

Page 29: The coefficient of $\frac{\partial z_n}{\partial p}$ in (4.2) should be

$$-[(a_n b_n)'_w(z_n-1)+a_n(b_n)'_w(2z_n-1)]$$

Page 32: a) In the statement of Lemma 4 is must be $\delta>\max(1;\frac{1-\lambda}{\lambda})$; b) In the last formula of the proof of Lemma 4 the constant M_1 should be replaced by $1+M_1$.

Page 33: The last formula in the proof of Proposition 1 must be replaced by

$$\frac{1}{F^{-1}(\varepsilon)}\int_0^{p_n} p^{\delta}(1-p)^{\delta}dp$$

Page 38: In the proof of Lemma 9 function must satisfy $0\le g(t)\le 1$ in I and

$$g(t)=\begin{cases} 1 & \text{if} \quad t\in\left(\frac{5t_0}{16}, \frac{7t_0}{16}\right), \\ 0 & \text{if} \quad t\notin I-\left(\frac{9t_0}{32}, \frac{15t_0}{32}\right). \end{cases}$$

8. The exact reference of [27] is: S. I. Shmarev, On the properties of interfaces for a class of degenerate parabolic equations of filtration theory. Dokladi Akademii Nauk, 326, no.3, (1992).

9. A manuscript incorporating the present modifications is available by request to some of the authors.

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